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CONTENTS

PAGE

J. W. PERRY. Thermal effects upon the performance of lens systems 257

E. W. H. SELWYN. The limit of visual resolution 286

B. K. JOHNSON. Recent optical materials and their possible applications 291

W. G. WEARMOUTH. Plastics and the optical industry 301

H. H. EMSLEY. Plastic spectacle lenses 314

Discussion : The resolution of optical images 321

G. N. PATCHETT. An application of the cathode-ray oscillograph to the measurement of the wave-length of sound 324

R. F. BARROW and E. E. VAGO. The band spectrum of SnSe in emission 326

H. T. FLINT and L. PINCHERLE. The impedance of hollow wave guides 329

E. F. HERROUN. A comparison of the magnetic properties of solid and powdered magnetites : with observations on coercive force 338

Reviews of books 344

Recent reports and catalogues 344

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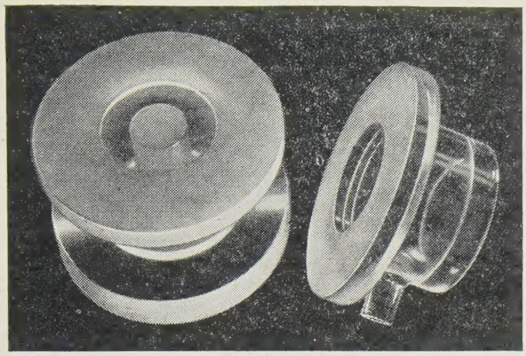

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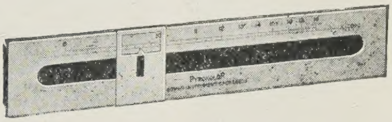

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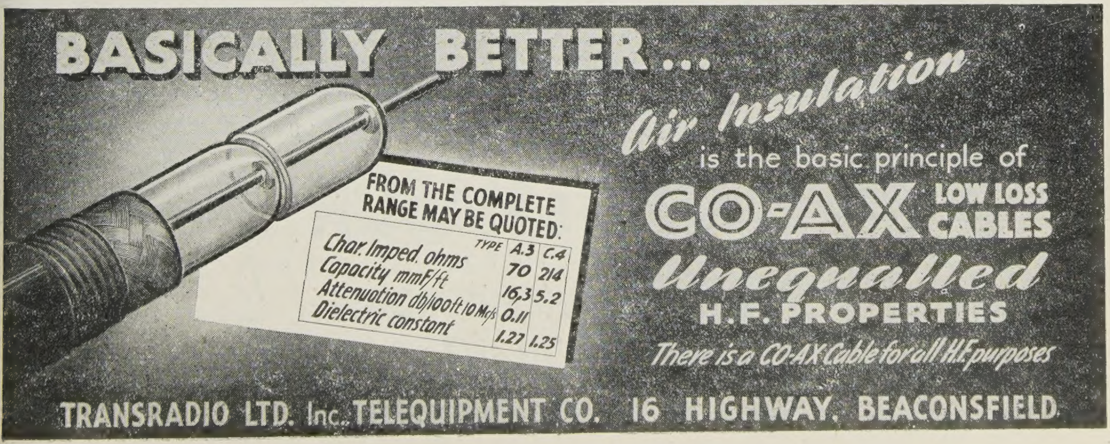

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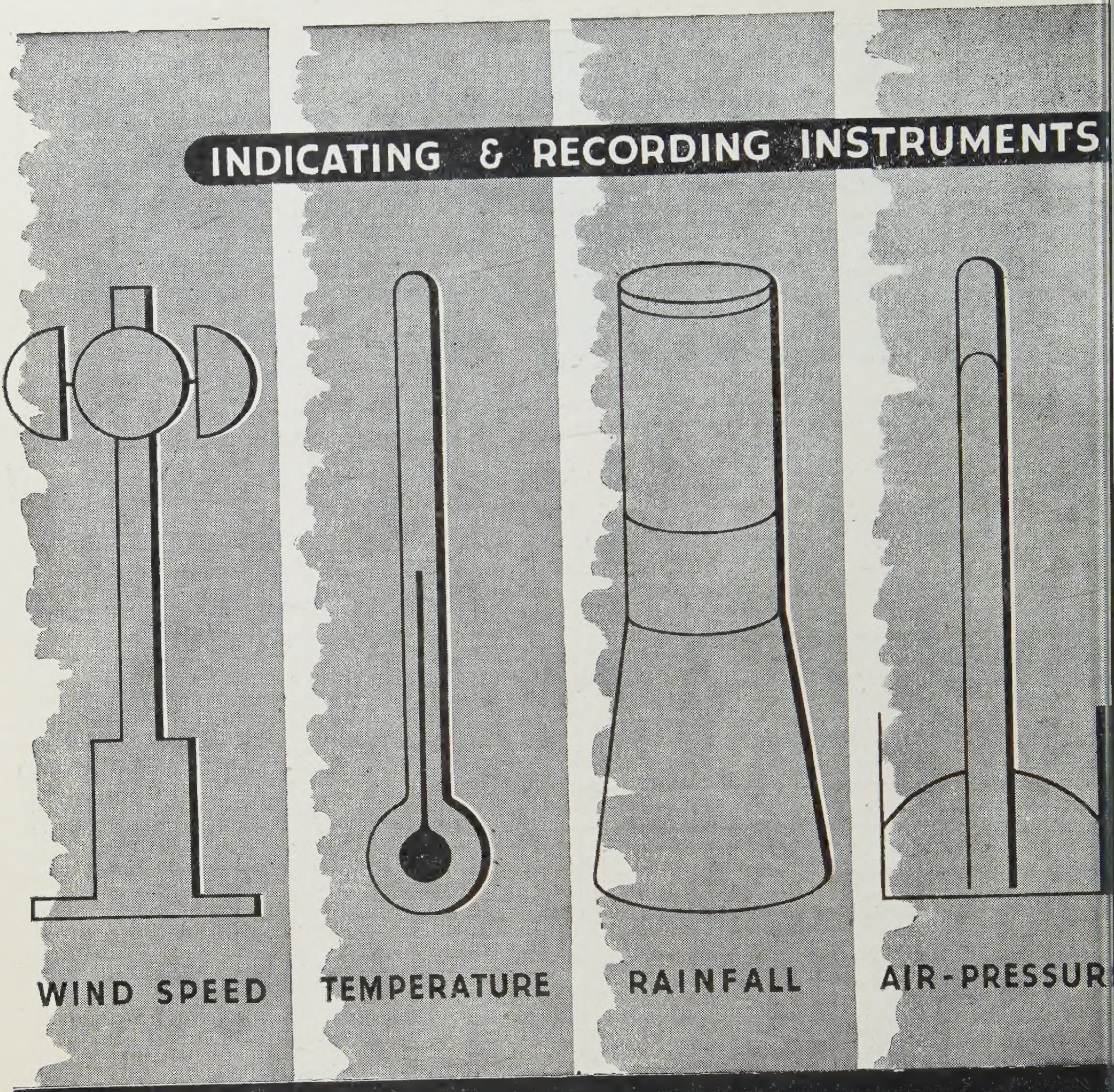

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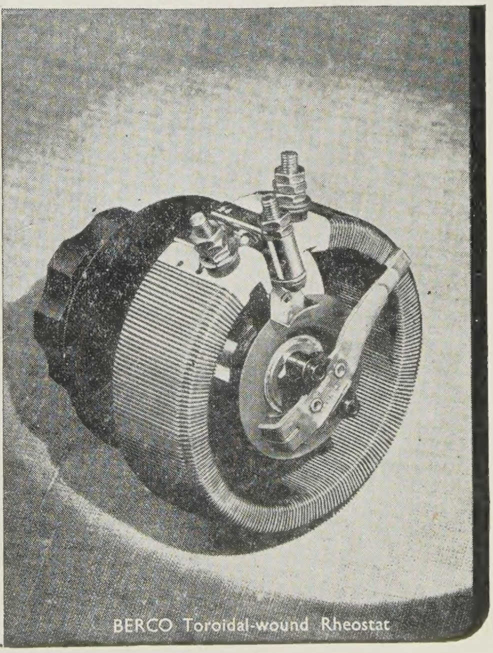

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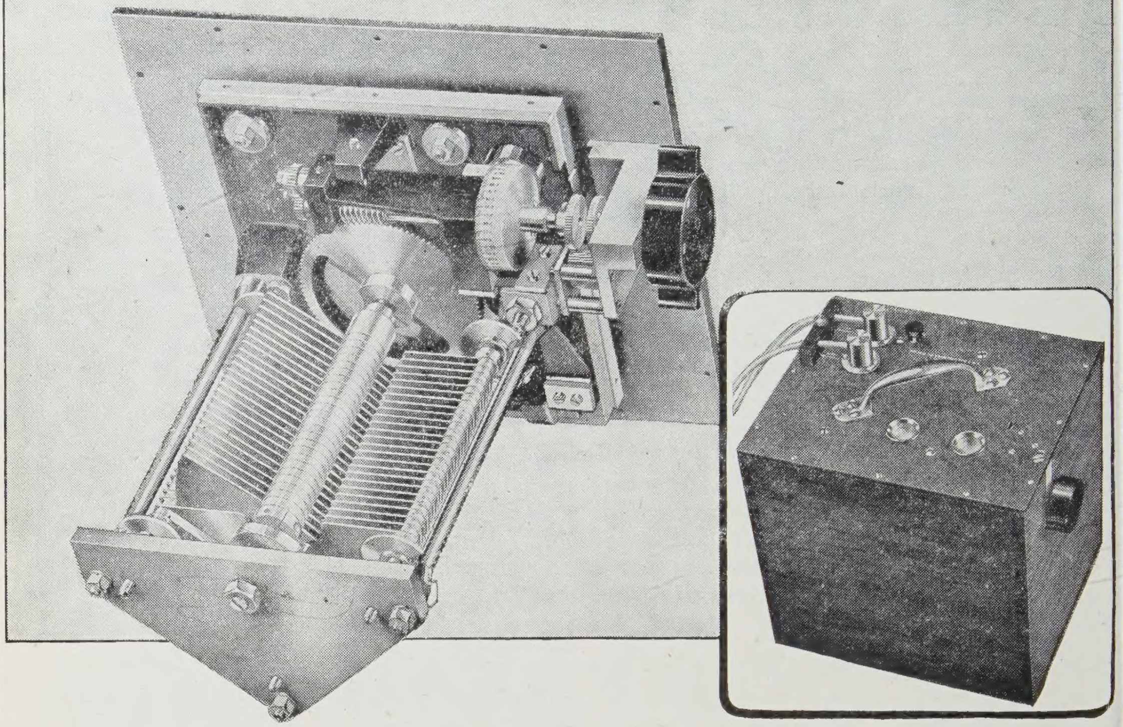

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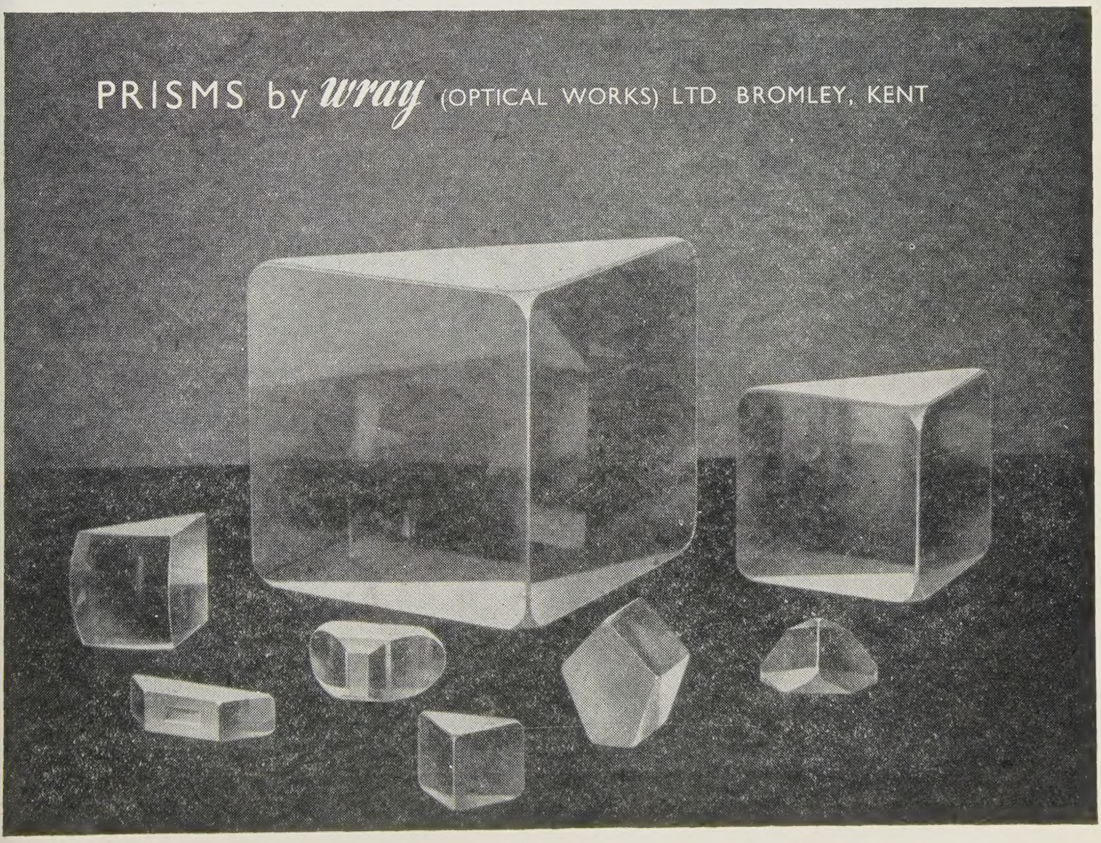

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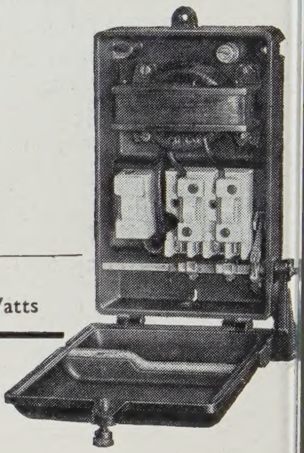

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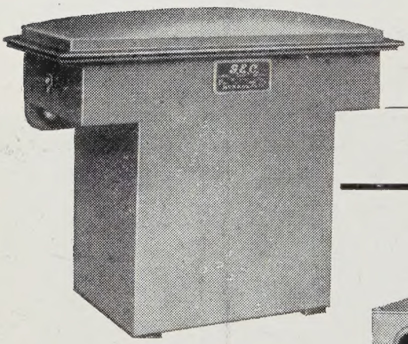

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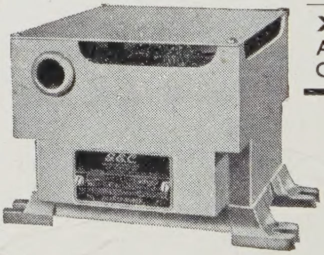

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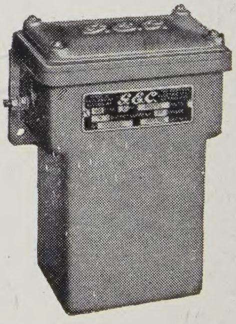

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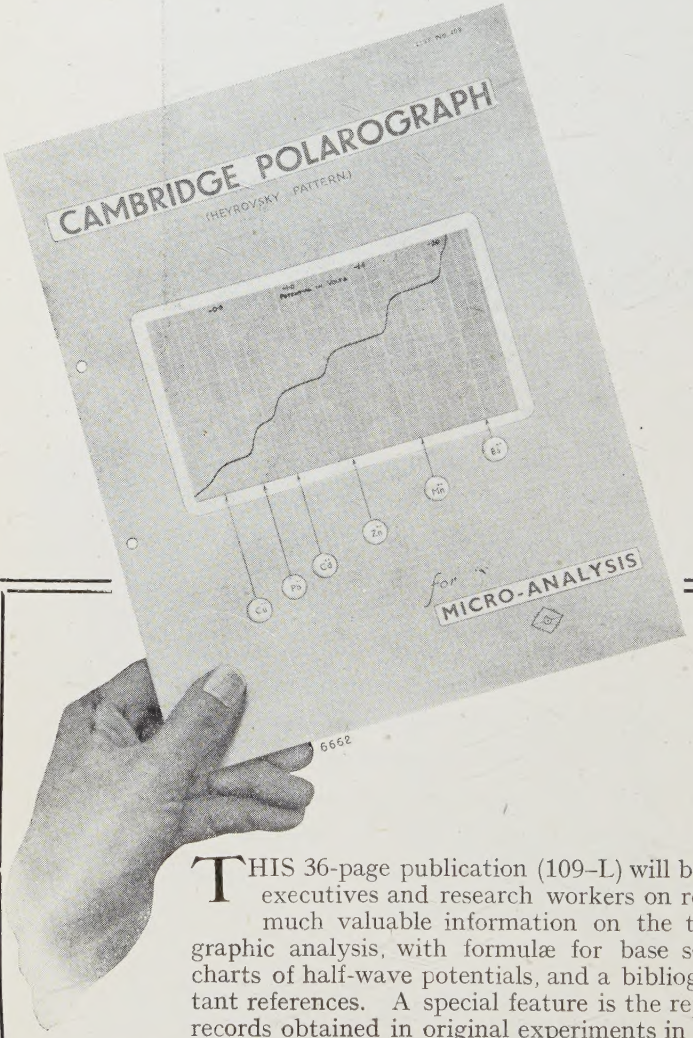

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THERMAL EFFECTS UPON THE PERFORMANCE OF LENS SYSTEMS

BY J. W. PERRY,
Adam Hilger, Ltd.

Paper read to the Optical Group 6 *March* 1942; *MS. received* 26 *January* 1943

ABSTRACT. The effects of heterogeneous temperature-distribution and uniform temperature change upon optical performance are considered in their general relationships and the practical problems of eliminating heterogeneous disturbances in optical instruments are reviewed. The quantitative effects of uniform temperature-variation upon image position and image size are evaluated in general terms as thermal aberrations. The conditions (*a*) for arbitrary thermal correction and (*b*) for thermo-mechanical correction, by which variations of tube length and image position are mutually compensated, are considered. The results are applied generally to the generalized doublet having one air space and any number of components; and applications to photographic and astronomical objective lens design are given in the form of examples of thermo-mechanical correction, calculated for two-lens systems with and without an air space, and for a combination of two doublets.

INTRODUCTION

THE performance of an optical system is known to be dependent upon properties which vary with temperature and is, in general, likewise variable with temperature. It may, however, also be affected indirectly and incidentally by diverse effects of thermal origin, producible during the course of the construction and use of an instrument. Such variations in optical performance—the effects, direct or indirect, of changes in the thermal conditions of materials—owe their existence mainly to the following circumstances and phenomena:

1. Variation in optical path length due to:

(*a*) Thermal expansion of the optical materials. The range of variation in thermal expansion covered by the various optical glasses and other materials available extends approximately from zero to 4.10^{-5}. For glasses the range is, however, relatively small.

(*b*) Thermal variation of refractivity. This may vary with varying materials between relatively distant limits, but in average cases is comparable, in its effect upon path length, with (*a*).

(*c*) Thermal expansion of mechanical mountings.

(*d*) Internal strain in optical media due to the stresses caused either during constructional operations or *in situ* by thermal expansions or contractions of the optical materials or the mechanical mountings.

2. Low thermal conductivity of optical materials, resulting in slow internal equalization of temperature.
3. Convection, and stratification in variations of density, in the atmosphere or other media surrounding the system.
4. Thermal variation of the transmission characteristics of spectrally selective components.
5. Thermal variation of the aberrational correction.

If we except 1 (*d*), which in practice may be controlled and largely eliminated by appropriate design, choice of materials and control of working processes, and defer consideration of (4), which is of limited application, it will be seen that such disturbing phenomena may be resolved into two broad classes, related each to a type of temperature variation characterizing the effects, as follows:

(i) A spatial thermal variation resulting in a heterogeneous temperature distribution, possibly variable with time but having a fixed mean value and extending, at any given time, possibly throughout the whole optical system, and

(ii) A secular thermal change, involving a variation of the mean temperature with time, the spatial departures from the mean being negligible throughout the optical system.

The "optical system" as here understood embraces the whole image-forming train of media extending from object to image.

In view of the direct dependence of optical performance upon temperature as a physical condition, it might appear at first sight to be somewhat anomalous that the optical effects of gross rise or fall in temperature customarily receive less attention than do the effects of comparatively slight heterogeneous temperature change. The explanation for this seeming anomaly will be apparent from the general conditions governing temperature effects in optical systems, which, in their turn, may be seen to follow from a simple consideration of the differential aspects of the phenomenon of image formation.

In order to determine the existence, in a practical sense, of an optical image, at a point Q, of an object at a point P, or vice versa, produced by a system free from palpable aberrational errors and suitably situated with reference to one of these points, we may, with negligible sacrifice to generality of reasoning, adopt here as a criterion that, over the whole finite aperture, A, of the image-forming system, the extent of the variation of the optical path between points P and Q through the system in question should not, irrespective of sign or analytical form, exceed a small quantity, ϵ, by which, in given conditions, the permissible limits of deterioration of the image would be determined. This may be expressed briefly in the form

$$\left| \Delta_{A} \sum_{P}^{Q} (nl) \right| < \epsilon.$$

Let P be considered fixed, let R be a point of variable position, and denote $\Delta_{A} \sum_{P}^{R} (nl)$ by the functional symbol η_R. Then, to state that P and Q are conjugate is tantamount to asserting that η_Q is a minimum value of η_R. In such case η_Q will consist primarily of two quantities, viz. (1) a variation, η_1, of optical path length, of a regular character, deducible from a knowledge of the aberrations of the system for any given temperature, and (2) a variation, η_2, in path length, due to temperature change of any kind. The additional suffix, Q, properly pertaining to these symbols, is here subintended, being omitted merely for convenience. A temperature variation, unrestricted in the form of its distribution, may now cause the position to be displaced at which the quantity η_R is a minimum, and the quantitative effect upon η_Q, resulting from such a redistribution of temperature, may consequently be *reduced* by appropriate adjustment of the position of Q. The *elimination* of such an effect will not, however, in general, be possible by any such adjustment, owing primarily to the heterogeneous effects. Apart, therefore, from thermal effects of a secondary and higher order which need not concern us here, one may thus distinguish in η_2 two parts, viz. (1) η_2', a "homogeneous" part, and (2) η_2'', a "heterogeneous" part, transferred epithets being used here in an obvious manner for verbal convenience.

From diffraction theory it is known to be futile to reduce ϵ to less than some quantity, ϵ_0 say, viz. some fraction of a wavelength of the image-forming radiation. The upper limit of ϵ is, in practice, fixed by such limits of performance as will satisfy stated requirements in given conditions. In order, however, that for a given object-point and optical system, and for a certain aperture, an image may be said to exist in the above sense, ϵ should be at least as great as η_1, the variation of path length due to zonal aberration, and should, in fact, exceed this by a certain permissible margin, $\delta\epsilon$ say, in order to provide for the effects of temperature variations.

If we assume a given system and specified conditions, then, to a given time-rate of change of temperature, will necessarily correspond, at a given time, a certain heterogeneous internal temperature distribution and corresponding local variations in path length. It follows that incipient homogeneous effects are inevitably accompanied by heterogeneous effects. In accordance with the foregoing, the local variations so occasioned may not exceed $\delta\epsilon$, which quantity therefore determines, for a given epoch and in given conditions, a certain limiting external temperature gradient, characteristic (*a*) of the system in question—particularly the masses of glass involved—and (*b*) of the degree of performance required.

When a high standard of definition or of resolution is called for, and $\epsilon - \epsilon_0$ requires in consequence to be less than or, at most, comparable with ϵ_0, the permissible margin will necessarily be small. In such a case, heterogeneous temperature change is clearly the factor which may decisively limit the performance by reason of its outstanding effect; any eventually perceptible displacement of focal position due to subsequently accruing homogeneous effects being then, clearly, of minor importance. It is necessary in such a case, in practice, to take steps to limit η_2'' appropriately, by employing means and methods of types subsequently to be referred to. When, on passing to systems of other forms and purposes, or owing to the diminishing effect of heterogeneous variations, either as a result of increased thermal diffusivity or reduced external temperature gradient, or for other reasons, the value of ϵ increases either absolutely or relatively, with the result that $\epsilon \geqslant (\eta_1 + \eta_2'')$, then the homogeneous temperature effects acquire practical significance, and in cases where temperature equalization is comparatively rapid, such effects may, in fact, entirely dwarf the heterogeneous effects. Temperature control will, of course, be the more difficult the greater the temperature gradient; so that, anticipating subsequent sections, it may, in some circumstances, be advantageous and preferable to annul the effect of temperature variations by design rather than to endeavour to maintain uniform temperature throughout the system.

The foregoing deductions ignore the refinements of exact theory, but may nevertheless suffice to co-ordinate the various factors involved and to present them in a reasonably true perspective. It would, evidently, be misleading to regard the thermal effects termed *heterogeneous* and *homogeneous* in the above as being each physically capable of spontaneous and independent occurrence. They are both, in fact, merely limiting forms, and their independent realization must depend essentially upon artificial means. Nevertheless, their frequent, apparently independent manifestation in practice, their different practical significance, and some important differences in the methods appropriate to their control and compensation, all justify their being considered for present purposes as independent phenomena.

§ 1. HETEROGENEOUS THERMAL EFFECTS

Various methods and devices may be resorted to in order to eliminate the effects of heterogeneous variations, chief among which the following may be mentioned:

1. Ventilation and artificial circulation of air in conjunction with artificial heating or cooling. It is important to admit such air circulation to both surfaces of large components, and, generally, to facilitate the maintenance of an equable temperature distribution by avoiding confined spaces in optical instruments.
2. Thermostatic control, especially if large masses are involved, or if the temperature coefficient of refraction is high, as in the case, for example, of most liquids and the denser flint glasses.
3. The use in instrument construction of an appropriate combination of non-conducting screens or covers with internal, conducting linings in order to reduce the influx of heat and to facilitate an equable distribution.
4. The use, when possible, of chosen optical materials with small temperature coefficients of refraction.
5. Obviating or minimizing the effect upon definition, when the major circumstances are beyond control, by arranging to use the system in such a manner that, although atmospheric heterogeneities remain, the isothermals will be perpendicular to the axis of the system.
6. To wait until a stationary thermal state is reached, and, in order that the period of waiting may not be indefinite or unduly prolonged, to influence appropriately the surrounding conditions.

General circumstances must, of course, be allowed to decide which one or more of these or other possible courses will be most appropriate and capable of effecting the desired degree of control; and in some cases, especially when long photographic exposure is necessary, this control may need to be of a very stringent character in order that the requisite standard of definition in the recorded image may be secured.

§ 2.1. HOMOGENEOUS THERMAL EFFECTS

In the course of this and subsequent sections, the numerical data for the thermal coefficients of linear expansion and refraction variation there appearing are stated in units of 10^{-5}. This constant factor is to be understood throughout, and, for general convenience, will be omitted from these coefficients. Only linear expansion will here be referred to. On some points as to nomenclature, see §6.

The effect upon optical performance of homogeneous temperature variation, inevitable and sometimes appreciable under the ordinary conditions of use of optical systems, may in some circumstances be of such magnitude as to limit satisfactory performance to certain thermal conditions. It is desirable then to observe special precautions in the design or use of systems. An obvious instance is when optical systems are called upon to function under varying conditions of temperature, but it may also quite easily happen when, for example, fixed-adjustment systems are required to function under dominating conditions such as geographic latitude or altitude, which differ significantly from those obtaining at the time of their adjustment. Reference to the effects of temperature difference upon optical performance have hitherto, to the writer's knowledge, been only of a qualitative character. Prof. A. E. Conrady (1917) has referred (verbally) to the serious effects of temperature change upon the performance of some lens systems.

The perceptible effects of uniform temperature change include primarily (*a*) displacement of the image with reference to the lens system and with reference to the operative image plane, and (*b*) variation of its dimensions. "Operative image plane" here signifies the plane at and upon which the image formed by the system is perceived or recorded, such as the photographic plate in the case of photographic systems. The accuracy of the ultimate image received will clearly depend upon the total variation of the position of the image with respect to the operative image plane.

As a simple illustration of a thermo-optical effect of this nature, consider the variation in the focal length, f, of a single lens, composed of an isotropic material and of negligible thickness, given by $1/f=(n-1)R$, n and R here denoting refractive index and total curvature. Logarithmic differentiation gives

$$-\frac{1}{f}\frac{df}{d\theta}=\frac{1}{n-1}\frac{dn}{d\theta}+\frac{1}{R}\frac{dR}{d\theta}.$$

The effect of linear expansion upon R may be considered to be that resulting from a thermal change in the unit of length, represented by a change in a general length modulus, $\boldsymbol{l}_\theta$, which represents the magnitude of the length unit as a function of temperature, θ. Then since, by dimensions, $R_\theta=R_0/\boldsymbol{l}_\theta$, where $\boldsymbol{l}_0=1$ and R_0, is, of course, a constant, it follows that, in general,

$$\frac{1}{R}\frac{dR}{d\theta}=-\frac{d}{d\theta}\log_e \boldsymbol{l}_\theta=-\alpha.$$

Hence

$$\frac{df}{d\theta}=f\left(\alpha-\frac{1}{n-1}\frac{dn}{d\theta}\right).$$

α here denotes the coefficient of linear thermal expansion. The coefficient of thermal variation of focal length, $\left(\alpha-\frac{1}{n-1}\frac{dn}{d\theta}\right)$, is, clearly, a physical constant characteristic of the optical medium, and in the following will be denoted by ϑ. Thus

$$\frac{df}{d\theta}=\vartheta f; \qquad \frac{d\phi}{d\theta}=-\vartheta\phi,$$

where $\phi=1/f$.

It may here be observed that, as will subsequently be evident, the thermal aberrations are formally similar in some respects to generalized chromatic aberrations. In spite of this we have, nevertheless, adopted for ϑ a sign the reverse of that following from this analogy, mainly out of consideration for some practical advantages, having reference to thermo-mechanical compensation, which are gained thereby and which will be clear from subsequent developments.

Table 1

Material	$\alpha.10^5$	$\vartheta_D.10^5$	$\nu\vartheta_D.10^5$
D.B.C. 610.574	+0·656	−0·016	− 0·92
E.D.F. 651.336	+0·875	+0·229	+ 7·7
B.S.F. 545.503	+0·939	+0·391	+ 19·7
D.F. 613.369	+0·908	+0·450	+ 16·6
L.F. 571.430	+0·893	+0·455	+ 19·6
B.S.C. 513.637	+0·802	+0·535	+ 34·1
M.B.C. 573.576	+0·800	+0·739	+ 42·6
B.L.F. 572.504	+0·876	+0·797	+ 40·2
H.C. 517.602	+0·912	+1·115	+ 67·2
Quartz (fused)	0·05	−2·13	−144·2
Fluorite	1·91	+4·68	+446·2
Rocksalt	4·04	+7·73	+330·8
Invar	0·15	—	—
Iron and average steels	1·05–1·25	—	—
Brasses	1·70–1·90	—	—
Aluminium	2·20–2·40	—	—

Expressions similar to those above may be deduced for lenses of crystalline materials, but in this case it is, of course, necessary to take account of (*a*) any dependence of the expansion coefficient upon orientation with respect to the crystalline axes, and (*b*) the possible dependence of the thermal variation of refractivity upon the state of polarization of the light. A simple case of some importance may here be mentioned in this connection, referring to the form of proof spheres made from materials such as crystalline quartz. The surfaces of such proof spheres are truly spherical only at some definite temperature, owing to the expansion coefficient being dependent upon orientation within the crystalline material. ϑ is, of course, dependent upon λ.

Some typical values of ϑ for λ_D for various materials, and other constants here referred to, are given in table 1 for *ca.* 20° c. In this table, and in what follows, values of α and of ϑ are expressed throughout in units of 10^{-5}. The data for glasses given in this table are based upon the work of Pulfrich (1892) and of Reed (1898), and also upon some information supplied directly by Messrs. Schott (Jena), to whom acknowledgement is due. The comparatively greater range covered by some other materials may be noted.

On the utilization of such data see §5.

Variations in the relative position of conjugate images in simple cases may be calculated from the values of ϑ above tabulated, but, needless to say, such values of ϑ, applying as they do merely to a single lens, convey only a partial and very inadequate impression of the magnitude of the effects of temperature upon image position for a compound system: in the case of combinations consisting of various components of differing materials, the use, as is customary, of comparatively high opposing powers does, in fact, result in a considerably enhanced effect. Moreover, to variations such as these, consisting of the purely thermo-optical effects, must be added similar effects of thermo-mechanical origin, resulting from the expansion or contraction of the lens mountings and the constructional framework of the instrument.

Table 2

Objective	λ	% thermal differences for ±50° c.				
		E.F.L. Al . $\mathrm{B^{SS}}$. Fe		B.F.L. Al . $\mathrm{B^{SS}}$. Fe		Tube length
Petzval portrait	G′	±0·112	±0·105	±0·118	±0·132	Al : ±0·118
Taylor triplet	G′	±0·023	±0·046	$\pm 0{\cdot}000_4$	±0·044	$\mathrm{B^{SS}}$: ±0·095
Fraunhofer doublet	D	±0·120		±0·120		Fe : ±0·055
Taylor photovisual	D	±0·32		±0·32		

Examples of calculated thermal variations of equivalent focal length and of back focal length, assuming the above-mentioned data, for some well known lens combinations, are tabulated in table 2, the percentage variation being given for a temperature change of ± 50° c. The lens mounting is assumed to be of the material indicated, viz. in one case aluminium ("Al", $\alpha = 2{\cdot}35$), in another brass ("$\mathrm{B^{SS}}$", $\alpha = 1{\cdot}9$), and in another of steel ("Fe", $\alpha = 1{\cdot}10$), thus covering the range of expansion commonly available. The corresponding percentage increment in the back vertex tube-length, assuming the same metals, is also given for purposes of comparison. Agreement of ΔBFL with tube-length difference is here, of course, desirable.

The Petzval portrait objective assumed is the original form detailed by v. Rohr (1901). The Taylor triplet is a form of the well known Cooke lens, approximately symmetrical in construction, as used, in the larger dimensions, for astronomical photography and aerial surveys, and employs glasses of the types D.B.C. and D.F. tabulated on p. 262. The photovisual objective is as specified by H. D. Taylor (1892). The doublet objective is the original combination of Joseph Fraunhofer (Stampfer, 1828). Components normally cemented have been assumed to be uncemented in the calculation of these quantities (*vide infra*).

The total variations here tabulated are related to the elementary variations above referred to, by formulae which we now proceed to deduce. It will be assumed that the operative image-plane is initially adjusted to coincide with the true image-plane for some specific temperature, the ultimately important quantity as regards image position being the relative displacement of true focal plane and operative image-plane.

§ 2.2. GENERAL THERMAL VARIATION OF IMAGE POSITION

The axial distance, s_i, from the ith vertex of the given system to the image plane in the adjoining medium is, in general, given by $s_i = s'_{i-1} - d_i$, where d_i denotes the axial separating distance between lens vertices. As a desirable innovation in notation, the axial thickness between surfaces $i-1$ and i of the lens component of refractive index $n'_{i-1} = n_i$ is here denoted correspondingly by d'_{i-1} or d_i. For present purposes d_i needs to be considered as the algebraic sum of axial lengths of the mechanical connections determining d_i. In general

$$\frac{ds'_{i-1}}{d\theta} - \frac{ds_i}{d\theta} = \frac{d\,d_i}{d\theta}.$$

In the case of object and image planes external to the system, we shall denote (*a*) the axial distance from the lens vertex in question to the preceding or following operative object or image plane by l or l' respectively, and (*b*) the distance from the operative image plane to the actual focal plane by ξ', so that $\xi' = s_k' - l'$. For the object plane the corresponding quantity is generally zero. In these cases, assuming a system of k refracting surfaces numbered in the direction of passage of the light, we shall then have

$$\left.\begin{array}{llll} (a) & \dfrac{dl}{d\theta} = Al = \dfrac{ds_1}{d\theta}\,; & (b) & \dfrac{dl_k'}{d\theta} = A'l'\,; \\[2ex] (c) & \dfrac{d\xi'}{d\theta} = \dfrac{ds_k'}{d\theta} - \dfrac{dl'}{d\theta}\,; & (d) & \dfrac{dd_i}{d\theta} = A_l d_l. \end{array}\right\} \quad \ldots\ldots(1)$$

A is here a constant for the total thermal expansion of l and is calculated from a knowledge of the members, numbered p, of the mechanical construction determining l, and the respective linear expansion coefficient, α, as follows:

$$A = \sum_p(\alpha_p l_p) / \sum_p(l_p). \qquad \ldots\ldots(2)$$

A' and $dd_i/d\theta$ are similarly determined.

The thermal variation of the back-vertex to image distance regardless of image formation, viz. $ds_k'/d\theta$, is an aberration in a general sense, and also needs to be known to enable the evaluation of $d\xi'/d\theta$.

§ 2.3. THERMAL VARIATION OF BACK-VERTEX IMAGE-DISTANCE

Given a centred system of k refracting surfaces, let the inclination to the optical axis of rays in an image-forming beam within the paraxial region be denoted by u, u' respectively before and after refraction at the boundary of radius r, between two isotropic media of refractive indices n, n' for the nominal wavelength λ.

Let Q_i denote the refraction invariant for the ith surface and Δ be the refraction difference operator. Then for a certain semi-aperture, h, by virtue of

$$\Delta(n/s)_i = \Delta(n/r)_i,$$

we have on differentiating with respect to θ:

$$\Delta\left(nu^2\frac{ds}{d\theta}\right)_i = \frac{h_i^2}{r_i^2}\frac{dr_i}{d\theta}\Delta(n)_i - h_i^2 Q_i \Delta\left(\frac{1}{n}\frac{dn}{d\theta}\right)_i, \qquad \ldots\ldots(3)$$

which applies, of course, to all refractions, $i = 1, 2 \ldots . k$. By virtue of

$$s'_{i-1} - s_i' = d'_{i-1} = d_i, \qquad \ldots\ldots(4)$$

and on substitution in (3) we get

$$n_i' u_i'^2 \frac{ds_i'}{d\theta} - n'_{i-1} u'^2_{i-1} \frac{ds'_{i-1}}{d\theta} = \left\{ \begin{array}{l} + h_i^2 \dfrac{n_i' - n_i}{r_i} \cdot \dfrac{1}{r_i}\dfrac{dr_i}{d\theta} - h_i^2 Q_i \Delta\left(\dfrac{1}{n}\dfrac{dn}{d\theta}\right)_i \\ - n'_{i-1} u'^2_{i-1} \dfrac{dd'_{i-1}}{d\theta}. \end{array} \right\} \qquad \ldots\ldots(5)$$

Recurrent expressions of this type apply to the sequence of refractions throughout the combination, and may be summed to relate the variation in image position to the constants of succeeding refractions, as follows:

$$\frac{ds_k'}{d\theta} = \left\{ \begin{array}{l} \dfrac{-1}{n_k' u_k'^2} \displaystyle\sum_{i=1}^{k} h_i^2 \left\{ Q_i \Delta\left(\frac{1}{n}\frac{dn}{d\theta}\right). - \boldsymbol{\varphi}_i \frac{1}{r_i}\frac{dr_i}{d\theta} \right\} \\ \dfrac{-1}{n_k' u_k'^2} \displaystyle\sum_{i=2}^{k} n_i u_i^2 A_i d_i + \frac{n_1 u_1^2}{n_k' u_k'^2}\frac{ds_1}{d\theta}. \end{array} \right\} \qquad \ldots\ldots(6)$$

$\boldsymbol{\varphi}_i$ is here written for $(n_i' - n_i)/r_i$, and A_i denotes the relative thermal increase in thickness in accordance with (2). In the case of a lens component or a simple spacing ring, $A_i = \alpha_i$, namely, the thermal-coefficient of linear expansion of the particular material employed. A_i is, in general, at one's disposal within the attainable limits, without affecting the optical system as such. As a physical constant characteristic of a given optical material, $dn/d\theta$ cannot, however, be varied except by a change of material which generally would affect the optical design.

The thermal variation of r. The quantity $dr_i/d\theta$ requires some consideration as, in the case of cemented lenses for example, it will depend largely upon the physical properties of the cement and also, possibly, upon the elastic properties of the optical media. When cementable contact surfaces are united, with or without the use of a cementing medium, in such a manner that relative movement between the surfaces is virtually impossible, the variation in radius will generally be determined by the properties of both components and be accompanied by axially symmetrical mechanical strain in the adjoining optical media, which will give rise to a defect of the *heterogeneous* class referred to above. Such errors strictly admit of no compensation, and the only remedy is, obviously, to dispense with such cementing processes when conditions are extreme, and to restrict their use to the range of temperature over which these effects would be practically insignificant. Outside this range, in cases when cementing is desirable, one may in favourable circumstances employ a "cement" which, with a limited rise or fall of temperature, will retain in sufficient degree the properties of fluidity or of viscous flow and thus be able to yield completely, although possibly with some time lag, to the stresses set up in the connecting medium. That such stresses may be considerable may readily be seen. In the case of a non-setting cement, the effects of surface tension may give rise to a traction of several atmospheres, varying appreciably with temperature, without detriment to the optical performance over an appreciable and ascertainable temperature range; whereas in a component lens of the same dimensions, contacted rigidly at the interface, the internal stresses that may arise with a relatively small change in temperature may be sufficient to render the combination entirely useless for the purpose intended.

The use of transparent non-setting cements capable of retaining properties of viscous flow over a specified temperature range does not provide a practical solution in the case of lenses of large dimensions; for, quite apart from the resulting problem involving the rheological and surface properties, unless the expansion coefficients of the cemented components be very closely similar, or the contact curvature relatively feeble, the thickness of the separating film, which under ordinary circumstances is necessarily small, would need to undergo a relatively large change at some part of the aperture, the result of which would be permanent rupture or contraction of the film. Special methods and devices for maintaining continuity in a liquid filling the interspace may be adopted in such cases, such as the rigid mounting at a suitable distance relatively to each other of the components to be so "cemented", and the incorporation of a reserve of such a connecting liquid, this being suitably chosen with regard to the relevant properties. Measures such as these, however, lend a much wider aspect to the purpose of dioptric cements in general; for the meniscus of cement, which in normal circumstances is optically negligible as a thin film, then acquires a thickness having a measurable effect and assumes the rôle of an independent liquid lens component. The commoner uses of cements form, of course, in this respect, but a special case.

Where cements are concerned we may therefore, from our present point of view, consider the more general case in which solid, assumedly rigid, media alternate with media which may be either viscous cements or fluid lens components. The factor $r_i^{-1} . dr_i/d\theta$ in (6) may in such case be denoted by $\alpha_{i,i'}$, which, as the fluid medium adjoining the ith surface is here able to yield to its changing form with varying temperature, will have the value applying to the rigid material forming that surface, viz. either α_i or α_i', according as the material either preceding or following the refraction is in the solid state. Each surface of such a fluid inter-lens must necessarily be considered independently, and a knowledge of n and $dn/d\theta$ for each such medium is therefore necessary.

The above expression for $ds_k'/d\theta$ may then be restated appropriately to this case, as follows:

$$\frac{ds_k'}{d\theta} = \left\{ \begin{array}{l} \dfrac{-1}{n_k' u_k'^2} \sum\limits_{i=1}^{k} h_i^2 \left\{ Q_i \Delta\left(\dfrac{1}{n}\dfrac{dn}{d\theta}\right)_i - \boldsymbol{\varphi}_i \alpha_{i,i'} \right\} \\ \dfrac{-1}{n_k' u_k'^2} \sum\limits_{i=2}^{k} n_i u_i^2 A_i d_i + \left(\dfrac{n_1 u_1^2}{n_k' u_k'^2}\right) \dfrac{ds_1}{d\theta} \end{array} \right. . \qquad \ldots\ldots(7)$$

It is to be noted here that while $\alpha_{i,i'}$ always refers specifically to optical media, the corresponding coefficient, A_i, represents, in accordance with the foregoing, a composite quantity which may relate either to mechanical or optical components or both.

The possibility of a time-lag in the attainment of a steady state following a temperature change depends upon the properties of the cement used. Limiting dimensions will depend largely upon the particular requirements as to definition and working conditions. Borderline cases will of course occur, and need to be studied from the point of view of securing the maximum possible alleviation by design or control, of the more extreme conditions. The major effects upon the calculated result of $ds_k'/d\theta$ of the presence of strain for small apertures will depend upon relative expansions at the cemented surfaces. As an approximation, an upper limit to this effect for any given contact between two solid media, i, $i+1$ say, may be found by calculating the quantity

$$\delta \frac{ds_k'}{d\theta} = -\frac{h_1^2}{2n_k' u_k'^2}(\alpha'_{i+1}-\alpha_i)(\varphi_{i+1}-\varphi_i).$$

Heterogeneous effects of a higher order will, of course, also be introduced at larger apertures if $(\alpha'_{i+1}-\alpha_i)$ be great, this being a point which may be borne in mind in the design of thermally stable systems, as the glass list includes a number of glasses of differing optical properties but with closely similar expansion coefficients. As affecting an important type of lens combination, mention may be made of the closeness of expansion coefficients of some types of hard crown and dense flint glasses.

The foregoing applies, of course, generally to systems of components without restriction as to thickness, and it is of some practical interest to reduce the above expressions to the form appropriate to systems of components of negligible thickness in air.

Assuming a combination of κ components ($\kappa=k/2$), we have as data for the jth lens component, bounded by surfaces i and $i+1$ say, where i is any odd numeral, and of power ϕ_j:

(*a*) $h_i=h_{i+1}=h_j$; (*b*) $\alpha_i'=\alpha_{i+1}$; (*c*) $n_i=1$;
(*d*) $n_i'=n_{i+1}$; (*e*) $n'_{i+1}=1$;

whence, by substitution in (7) and after some rearrangement, the following is obtained as applying to separated lens components of negligible thickness:

$$u_k'^2 \frac{ds_k'}{d\theta} = +\sum_{j=1}^{\kappa} h_j^2 \vartheta_j \phi_j - \sum_{j=2}^{\kappa} u_j^2 A_j d_j + u_1^2 \frac{ds_1}{d\theta}. \qquad \ldots\ldots(8)$$

If the system in question consist of κ contact-combinations of such components, each combination being numbered serially μ, the components of each combination being numbered λ and air spaces numbered ν, we have analogously, neglecting the variation $ds_1/d\theta$,

$$u_\kappa'^2 \frac{ds_\kappa'}{d\theta} = +\sum_{\mu=1}^{\kappa} h_\mu^2 \sum_{\lambda} \vartheta_{\mu\lambda}\phi_{\mu\lambda} - \sum_{\nu} u_\nu^2 A_\nu d_\nu. \qquad \ldots\ldots(9)$$

From these expressions one may perceive directly the modifying effect upon the contribution, due to the coefficient ϑ, of the thermal expansion of the air spaces. Unless the vergency of the beam in a given air space be relatively small, the magnitude of this part of the variation will be of an importance comparable with the part due to ϑ.

§ 2.4. THERMAL VARIATION OF TRANSVERSE LINEAR MAGNIFICATION

Some additional symbols are here required, as follows:

t'_{i-1}, t_i: the axial intersection distances of the principal ray between surfaces $i-1$ and i, measured from the respective vertices of these surfaces.

v'_{i-1}, v_i: the corresponding obliquities to the optical axis.

y'_{i-1}, y_i: the corresponding surface intersection heights, measured from the optical axis.

$M_i = s_i - t_i$,

$e_i = y_i/h_i$,

$T = n_1 u_1 v_1 M_1$.

Consider again the previous system of k refracting surfaces forming, for some given temperature, $\theta = 0$ say, an image at the focal plane, distant s_k' from the kth surface. The linear magnification, $\beta'_{1,k}$, at the operative focal plane, situated, in accordance with the above, at distance l' from the kth surface, and not necessarily identical with the actual focal plane, is given by

$$\beta'_{1,k} = \frac{n_1}{n_k'} \frac{l' - t_k'}{M_k'} \prod_{i=1}^{k} \left(\frac{s_i'}{s_i}\right). \qquad \ldots\ldots(10)$$

For $\theta = 0$ it is assumed that, as the result of an adjustment, the image formed at s_k' is focused on the plane at l', so that $\xi' = s_k' - l' = 0$. From (10) we may derive

$$\frac{1}{\beta'_{1,k}} \frac{d\beta_{1,k}}{d\theta} = + \sum_{i=2}^{k} \left(\frac{1}{s'_{i-1}} - \frac{1}{s_i}\right) \frac{ds'_{i-1}}{d\theta} + \sum_{i=2}^{k} \frac{1}{s_i} \frac{dd_i}{d\theta} + \left(\frac{1}{n_1} \frac{dn_1}{d\theta} - \frac{1}{n_k'} \frac{dn_k'}{d\theta}\right)$$
$$- \left(\frac{1}{s_1} \frac{ds_1}{d\theta} - \frac{1}{s_k'} \frac{ds_k'}{d\theta}\right) - \frac{1}{M_k'} \frac{d\xi'}{d\theta}. \qquad \ldots\ldots(11)$$

This is inconvenient for calculation, particularly in that it contains the intermediate s and $ds/d\theta$ values. The procedure generally followed in analogous cases for the elimination of these values is somewhat circumlocutory and may be avoided, for, by virtue of the readily proven relation

$$\frac{M_i}{s_i} - \frac{M'_{i-1}}{s'_{i-1}} = \frac{t'_{i-1}}{s'_{i-1}} - \frac{t_i}{s_i},$$

(11) may be expressed in the form

$$\frac{1}{\beta'_{1,k}} \frac{d\beta'_{1,k}}{d\theta} = - \sum_{i=1}^{k} \Delta \left(\frac{t}{Ms} \frac{ds}{d\theta}\right)_{.i} + \sum_{i=2}^{k} \frac{1}{M_i} \frac{dd_i}{d\theta} + \left(\frac{1}{n_1} \frac{dn_1}{d\theta} - \frac{1}{n_k'} \frac{dn_k'}{d\theta}\right)$$
$$- \frac{1}{M_k'} \frac{d\xi'}{d\theta} - \left(\frac{1}{s_1} \frac{ds_1}{d\theta} - \frac{1}{s_k'} \frac{ds_k'}{d\theta}\right) - \left(\frac{t_1}{M_1 s_1} \frac{ds_1}{d\theta} - \frac{t_k'}{M_k' s_k'} \frac{ds_k'}{d\theta}\right),$$

which reduces to

$$\frac{1}{\beta'_{1,k}}\frac{d\beta_{1,k}}{d\theta} = +\frac{1}{T}\sum_{i=1}^{k} e_i\Delta\left(nu^2\frac{ds}{d\theta}\right)_i + \sum_{i=2}^{k}\frac{1}{M_i}\frac{dd_i}{d\theta} + \left(\frac{1}{n_1}\frac{dn_1}{d\theta} - \frac{1}{n_k'}\frac{dn_k'}{d\theta}\right)$$
$$-\left(\frac{1}{M_1}\frac{ds_1}{d\theta} - \frac{1}{M_k'}\frac{ds_k'}{d\theta}\right) - \frac{1}{M_k'}\frac{d\xi'}{d\theta}.$$

As the first term on R.H.S. is known by (3) above, we finally obtain

$$\frac{1}{\beta'_{1,k}}\frac{d\beta'_{1,k}}{d\theta} = \left\{\begin{aligned} & -\frac{1}{T}\sum_{i=1}^{k} e_i h_i^2\left\{Q_i\Delta\left(\frac{1}{n}\cdot\frac{dn}{d\theta}\right)_i - \varphi_i\frac{1}{r_i}\frac{dr_i}{d\theta}\right\} + \sum_{i=2}^{k}\frac{A_i}{M_i}d_i \\ & + \left(\frac{1}{n_1}\frac{dn_1}{d\theta} - \frac{1}{n_k'}\frac{dn_k'}{d\theta}\right) - \left\{\frac{1}{M_1}\frac{ds_1}{d\theta} - \frac{1}{M_k'}\frac{ds_k'}{d\theta}\right\} - \frac{1}{M_k'}\frac{d\xi'}{d\theta},\end{aligned}\right\} \qquad \ldots\ldots(12)$$

where $ds_k'/d\theta$ is given by (7) above.

As in the foregoing, and subject, of course, to the same restrictions in regard to contiguous surfaces, $r_i^{-1}\,dr_i/d\theta$ may here likewise be replaced by $\alpha_{i,i'}$. In a manner also similar to that of the foregoing, we may obtain for a system of lenses of negligible thickness in air:

$$\frac{1}{\beta'_{1,\kappa}}\frac{d\beta'_{1,\kappa}}{d\theta} = -\frac{1}{T}\sum_{j=1}^{\kappa} e_j h_j^2\vartheta_j\phi_j + \sum_{j=1}^{\kappa}\frac{A_j}{M_j}d_j - \left\{\frac{1}{M_1}\frac{ds_1}{d\theta} - \frac{1}{M_\kappa'}\frac{ds_\kappa'}{d\theta}\right\} - \frac{1}{M_\kappa'}\cdot\frac{d\xi'}{d\theta}, \qquad \ldots\ldots(13)$$

where, in the evaluation of this formula, $ds_\kappa'/d\theta$ would be given by (8).

Thermal errors of this kind may, in photogrammetry for example, need to be considered in conjunction with the thermal variation in the size of the receiving and recording surface. Their magnitude depends mainly upon the terms in $ds_1/d\theta$ and $ds_\kappa'/d\theta$, which are, of course, also involved in the condition for the correction of $d\xi'/d\theta$. The reduction of image dimensions to some standard temperature may be effected as in the following. This, however, involves corrections of quite small magnitude and is rarely likely to be of practical importance.

If Y_θ denote the length of an actual line image and $\boldsymbol{Y}_\theta$ the length of the corresponding recorded image, both at temperature θ, then to reduce the length, $\boldsymbol{Y}_2$, of the image recorded at temperature θ_1 and measured at temperature θ_2, to the image size, Y_0, at some conventional temperature, θ_0,

$$Y_0 = \boldsymbol{Y}_2\left\{1 + (\theta_0 - \theta_1)\frac{1}{\beta'_{1,k}}\frac{d\beta'_{1,k}}{d\theta} + (\theta_1 - \theta_2)\alpha\right\},$$

where α here denotes the expansion coefficient of the image-recording material.

§ 3.1. THERMAL ABERRATION AND CORRECTION: GENERAL OBSERVATIONS

(*a*) *Longitudinal aberration*

It is desirable here to distinguish between (*a*) general thermal correction, by which the regular thermal properties of image-forming systems are controlled and corrected to comply with some prescribed condition, and (*b*) thermo-mechanical correction, which aims at reducing to zero the thermal displacement of the image from some particular operative focal plane. The latter is, of course, the more important from the point of view of the correction of complete systems. The condition for this is obtained by equating $d\xi'/d\theta$ to zero, viz.,

$$\frac{d\xi'}{d\theta} = 0. \qquad \ldots\ldots(14)$$

to which $(1(c))$, (6), (7) and (8) are subsidiary. In evaluating $ds_k'/d\theta$ therein, it may be noticed that the part due to thickness, for example $-\sum_i n_i u_i^2 A_i d_i$ in (6), is always negative (unless, by the use of an over-compensating combination of materials, A_i be made negative). The effect of this term is generally to reduce the purely thermo-optical effect due to the summations involving ϑ or corresponding terms. The most generally applicable method of correcting longitudinal thermal aberration is, clearly, by compensating the purely thermo-optical effects by the effects of thermo-mechanical origin.

(*b*) *Transverse aberration*

The general condition for the correction of thermal magnification errors is provided by equating (12) or (13) to zero. The purely thermo-optical contributions to the thermal variation of magnification have here no characteristic sign, and, in the case of most combinations, unless these are entirely devoid of the extended symmetry material to the correction of the monochromatic aberrations, some measure of internal compensation will result between terms of the same type, as a result of a diffuse connection which appears to exist between ϑ and ν. Longitudinal correction requires that $d\xi'/d\theta$ be reduced to within satisfactory limits, and prior consideration would in practice need to be given to this. Thus, for the majority of systems the thermal error of magnification will consist very largely of the part due to

$$\frac{1}{M_1}\frac{ds_1}{d\theta} - \frac{1}{M_{\mathrm{K}}'}\frac{ds_{\mathrm{K}}'}{d\theta}.$$

Under certain special conditions, these two terms may be made to compensate each other.

It may be mentioned that the above conditions have a constructive value in relation to already existing systems in that they may, in some cases, be applied in order to secure correction by the introduction of appropriate compensating thermo-mechanical variations without these affecting the existing correction of other aberrations. This refers particularly to longitudinal thermal aberration, to which thermal aberration our attention in the following will now be confined.

§ 3.2. APPLICATIONS TO COMPOUND SYSTEMS WITH ONE AIR-SPACE: THERMAL ABERRATION AND CORRECTION

On the basis of the foregoing, we now proceed to consider, for systems of two axially-separated *members*, each consisting of one or more *components* in contact, the general question of the correction of longitudinal thermal aberration for incident parallel light, the relation between such thermal aberration and the corresponding chromatic aberration, and the possibility of securing simultaneous correction of both these aberrations in systems providing basic forms for some important types of astronomical and photographic objectives.

We shall consider first the case of the correction of thermal aberration in the general sense, this having applications particularly to partial systems and, subsequently, the special case of the thermo-mechanical correction of complete systems.

(i) *Thermal correction*

The members of the system will be numbered serially, i, and, for each member, the component single lenses, assumed to be of negligible thickness, will be numbered serially, j, enumeration proceeding in each case in the direction of the incident light. As applying to the ith member of such a system and determining its power and states of thermal and chromatic correction respectively, we have

$$\sum_j \phi_{ij} = \phi_i, \qquad \text{......(15)}$$

$$\sum_j \vartheta_{ij}\phi_{ij} = \Theta_i'\phi_i, \qquad \text{......(16)}$$

$$\sum_j \varpi_{ij}\phi_{ij} = w_i\phi_i, \qquad \text{......(17)}$$

in which $\varpi_{ij} = 1/\nu_{ij}$ and (16) follows from (8). The quantities Θ_i and w_i, as so defined, determine the corresponding aberrations present in each member. If member i is a single lens, then $\Theta_i = \vartheta_{i1}$ and $w_i = \varpi_{i1}$.

Similarly, for a system of two axially-separated compound members,

$$h_1\phi_1 + h_2\phi_2 = h_1\Phi, \qquad \text{......(18)}$$

$$h_1^2\bar{\Theta}_1\phi_1 + h_2^2\Theta_2\phi_2 = h_1^2\Theta\Phi, \qquad \text{......(19)}$$

$$h_1^2w_1\phi_1 + h_2^2w_2\phi_2 = h_1^2W\Phi, \qquad \text{......(20)}$$

where $\bar{\Theta}_1 = \Theta_1 - \alpha_1'(1-h_2/h_1)$, in which α_1' is the linear expansion coefficient for the air separation. In this and the ensuing sections, α_1' will be continuously variable and will denote the effective overall expansion coefficient. It will thus take the place of A_1' in the foregoing. We shall, moreover, in the following, denote h_2/h_1 by q. The mechanical tube-length will be assumed equal to the back focal-length. From (18) and (19) follows, for general thermal correction without regard to other conditions,

$$\left.\begin{aligned} \phi_1 &= \Phi\left\{\frac{q\Theta_2-\Theta}{(q\Theta_2-\Theta_1)+\alpha_1'(1-q)}\right\}, \\ q\phi_2 &= \Phi\left\{\frac{(\Theta_1-\Theta)\ -\alpha_1'(1-q)}{(\Theta_1-q\Theta_2)-\alpha_1'(1-q)}\right\}. \end{aligned}\right\} \qquad \text{......(21)}$$

The relation (22), subsisting between the coefficients in (18), (19) and (20), assuming $\Phi = 1$ and $h_1 = 1$, enables us to deduce the magnitude of one aberration, W or Θ, when the other is zero or is present in a stated amount, both for complete systems, with or without an air space, and for the combinations forming the members of the same:

$$\begin{vmatrix} +1 & +1 & -1 \\ w_1 & qw_2 & -W \\ \bar{\Theta}_1 & q\Theta_2 & -\Theta \end{vmatrix} = 0. \qquad \text{......(22)}$$

It will often be more appropriate to regard the thermal error as dependent upon other conditions. We readily find the thermal aberration characteristic, Θ, as a function of W and the remaining variables, to be as follows:

$$\Theta = \frac{q(w_1\mathit{\Theta}_2 - w_2\mathit{\Theta}_1) + W(\bar{\mathit{\Theta}}_1 - q\mathit{\Theta}_2)}{(w_1 - qw_2)}, \qquad \ldots\ldots(23)$$

where w_1 and w_2 may have any values predetermined by virtue of (17) and are respectively equal to $1/\nu_1$ and $1/\nu_2$ if the members are each single lenses. As a sub-case to the form which this expression assumes if $q = 1$ and $w = 0$, we have, for a system of two single members in contact, and possibly cementable, forming a simple, achromatic, doublet objective,

$$\Theta_{(q=1,\, W=0)} = (\nu_2\vartheta_2 - \nu_1\vartheta_1)/(\nu_2 - \nu_1), \qquad \ldots\ldots(24)$$

which quantity we shall in the following denote by Θ'', or, in the case of partial systems, $\mathit{\Theta}''$. Corresponding expressions for a system of three or more members in contact may similarly be derived from the above, considered in relation to any other conditions which may apply.

Table 3. Values of Θ'' for various contact-doublet glass combinations. (By § 3.2 (ii) this is also a table of values of $\alpha_0{}'$ for separated doublets.)

−ve +ve component	B.L.F. 572.504	B.S.F. 545.503	L.F. 571.430	D.F. 613.369	E.D.F. 651.336
B.S.C. 513.637	−0·46	1·07	0·70	0·65	0·88
H.C. 517.602	2·75	4·79	2·76	2·17	2·24
M.B.C. 573.576	—	3·14	1·58	1·25	1·45
D.B.C. 610.574	—	—	−1·42	−0·86	−0·36
B.L.F. 572.504	—	—	—	1·74	1·93

Values of Θ'' are tabulated for pairs of glasses in table 3 and the outstanding thermal aberration of the simple achromatic doublet and its range of variation for the more serviceable combinations of materials may be perceived therefrom. The condition for $\Theta'' = 0$ (assuming $W = 0$), which is, clearly, $\nu_1\vartheta_1 = \nu_2\vartheta_2$, will be seen from table 3 to be not nearly realizable in a doublet for the commoner pairs of materials, but the need for this is not likely to arise, except possibly in order to satisfy specific subsidiary conditions relating to partial systems.

The correction of aberrations does not, of course, always or necessarily require the exact annulment of quantities such as W or Θ, for, as was previously mentioned, Θ will need to have a certain finite value to secure thermo-mechanical correction, and in the case of some spectrographic and other systems, W may also be relatively large. In this respect we may regard the above as providing conditions to be satisfied in conjunction with (19) or (20), in order that any desired adjustment may be made in the values W and Θ simultaneously, and also in order that thermal correction may be secured in the special sense of (14).

The relation (22) may be applied in various alternative forms appropriate to the purposes of calculating either α_1' or q, assuming the aberrations W and Θ for the whole system to be prescribed, as follows:

$$q^2 . w_2\alpha_1' + q[w_2(\Theta_1 - \Theta) - \Theta_2(w_1 - W) - \alpha_1'(w_2 + W)] - \{(\Theta_1 W - w_1\Theta) - W\alpha_1'\} = 0, \qquad \ldots\ldots(25)$$

and a corresponding form for α_1'. The value of q, if not determined in this manner, may, of course, be made dependent upon other conditions. From a knowledge of q we may find ϕ_1 and ϕ_2, as follows:

$$\left.\begin{aligned} \phi_1 &= \Phi\left\{\frac{qw_2 - W}{qw_2 - w_1}\right\}, \\ q\phi_2 &= \Phi\left\{\frac{w_1 + W}{w_1 - qw_2}\right\}. \end{aligned}\right\} \qquad \ldots\ldots(26)$$

When the members of a system are contact doublets, the values of Θ_i for substitution in (25) or (26) follow from table 3 if $w_i = 0$; or may be derived therefrom if $w_i \neq 0$, by the addition of a correcting term based upon (23), giving in this case

$$\Theta_i = \Theta_i'' + w_i \frac{\vartheta_{i1} - \vartheta_{i2}}{w_{i1} - w_{i2}}. \qquad \ldots\ldots(27)$$

Otherwise (23) may be used to effect the elimination of W_i or Θ_i from the equations under consideration.

(ii) *Thermo-mechanical correction*

If the true image-plane and the operative image-plane are to remain identical with change of temperature, then, as indicated above, $d\xi'/d\theta = 0$. Let Θ' be defined as follows:

$$\frac{d\xi'}{d\theta} = \frac{1}{u_k'^2} h_1^2 \Theta' \Phi. \qquad \ldots\ldots(28)$$

Then by 1 (*c*), for unit Φ,

$$\frac{1}{u_k'^2} h_1^2 \Theta' = \frac{ds_k'}{d\theta} - \alpha_2' l', \qquad \ldots\ldots(29)$$

which, for incident parallel light, gives

$$\Theta = \Theta' + q\alpha_2'. \qquad \ldots\ldots(30)$$

Introducing this value for Θ into (19), we find

$$h_1^2 \bar{\Theta}_1 \phi_1 + h_2^2 \Theta_2 \phi_2 = h_1^2(\Theta' + q\alpha_2'), \qquad \ldots\ldots(31)$$

which determines Θ', or which, on putting $\Theta' = 0$, provides the condition for thermo-mechanical correction (abbreviated in the following to "θ-m. correction") in the sense of the annulment of Θ' and, therefore, of $d\xi'/d\theta$. Thus, if $d\xi'/d\theta = 0$, we find (*a*) in general, for a contact system of single lenses, ($q = 1$), and (*b*) in the simplest possible case of a simple, achromatic, contact doublet, used with a telescope body composed possibly of one kind of material throughout, respectively:

$$\left.\begin{aligned} &(a) \quad \Theta = \alpha_2' \equiv A' \text{ by (2)}, \\ &(b) \quad \Theta'' = \alpha_2'. \end{aligned}\right\} \qquad \ldots\ldots(32)$$

On comparing Θ'' as given in table 3 with the expansion coefficient for some materials, given at the foot of table 2, it will be seen that, for a simple achromatic doublet, thermo-mechanical correction may readily be obtained by a suitable choice of lens and instrument materials.

If in the equations (22) to (25) we substitute $(\Theta' + q\alpha_2')$ for Θ by (30), we shall now obtain corresponding forms relating to θ-m. correction in place of general thermal correction, and analogous considerations will apply. Some of the more interesting and important results and conclusions in this respect will here be reviewed.

When chromatic and θ-m. corrections are both stipulated, the resulting condition, following from the above substitution for Θ, may be stated in the form

$$\alpha_1'(1-q)(qw_2 - W) + \alpha_2' q(qw_2 - w_1) = q(w_1\Theta_2 - w_2\Theta_1) - W(\Theta_1 - q\Theta_2). \qquad \ldots\ldots(33)$$

Thus, if q is known, a range of solutions for combined chromatic and θ-m. corrections, consisting of pairs of correlated values of α_1' and α_2', may be derived, and such values may be mutually adjusted in suitable conditions to suit practical convenience.

In order to obtain a clearer conception of the general form of the relationship existing between α_1' and α_2' in (33) we may regard the W terms merely as corrections and, leaving their modifying effect to be considered subsequently, put $W = 0$. By this we obtain for such a chromatically corrected system

$$\left(\frac{w_2}{w_1} - 1\right)\alpha_0' + (1-q)\alpha_1' + \left(q - \frac{w_2}{w_1}\right)\alpha_2' = 0, \qquad \ldots\ldots(34)$$

in which

$$\alpha_0' = \frac{w_1\Theta_2 - w_2\Theta_1}{w_1 - w_2}. \qquad \ldots\ldots(35)$$

(34) represents, in rectangular co-ordinates $(\alpha_1', \alpha_2', q)$, a double, right conoid whose axis is perpendicular to the (α', α_2') plane and intersects this plane in the point (α_0', α_0'). It may be represented by a projection on the (α_1', α_2') plane, of the generating line in various selected positions relating to specific values of q, as shown in figure 1. From this it may be seen that a form of symmetry about α_0' exists between the values α_1' and α_2'. This is exhibited more clearly by referring (34) to (α_0', α_0') as origin, as follows:

$$(1-q)(\alpha_1' - \alpha_0') + \left(q - \frac{w_1}{w_2}\right)(\alpha_2' - \alpha_0') = 0, \qquad \ldots\ldots(34\,a)$$

a form which is of value when mutually adjusting the α_1', α_2' values. The range of q values interesting in the present connection extends from 1 to w_1/w_2 (i.e., in the case of single members, 1 to ν_2/ν_1), but practical restrictions prevent a near approach to the latter limit.

As is evident from the foregoing, $(\alpha_1' = \alpha_0', \alpha_2' = \alpha_0')$ constitutes a solution which applies whatever value q may have. It may be noted that this solution for thermo-mechanical correction has an autogenous character in regard to the general physical properties involved, as the expansion coefficients of the lens mountings and of the body of the telescope or camera are thus determined uniquely and solely by the intrinsic properties of the component lens materials, regardless of the power or dimensions of the system or of its members. The value, α_0', so found for the expansion coefficients of the mountings of a system of two *separated* members is, incidentally, equal to Θ'', the thermal aberration characteristic for a *contact* combination of the same materials. Thus, for any given pair of materials, α_0' may, consequently, also be taken at sight from table 3. It will not, of course, always be convenient to adopt this autogenous solution for a given system under design, and alternative solutions will need to be considered, using (34 *a*). This will generally be found to provide a suitable pair of values unless, by reason of the choice of optical materials, the α_0' value be too extreme.

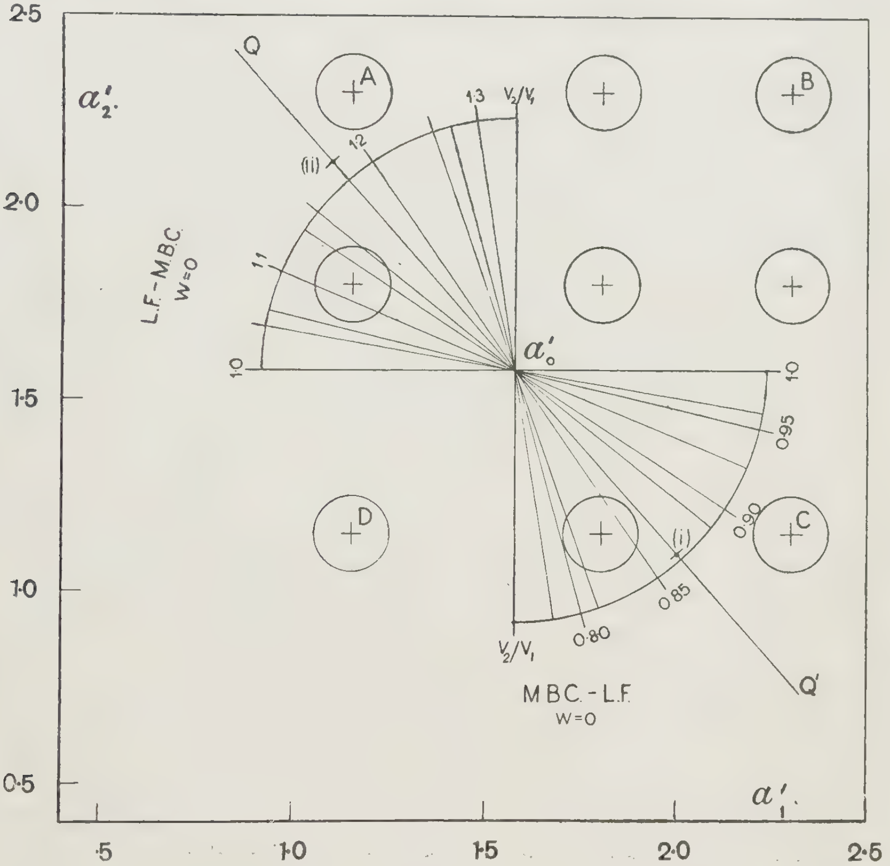

Figure 1.

When, in an achromatized system, the members are also individually achromatic, ($w_1 = 0$, $w_2 = 0$), such expressions become inapplicable, for ϕ_1 and ϕ_2 are then independent of (20). We may then, however, derive directly from (19) an alternative relation, parallel to the above, viz.,

$$\phi_1(1-q)(\alpha_1' - \alpha_0'') + q(\alpha_2' - \alpha_0'') = 0, \qquad \ldots\ldots(36)$$

where

$$\alpha_0'' = \frac{(\Theta_1\phi_1 + q^2\Theta_2\phi_2)}{(1-q)\phi_1 + q}. \qquad \ldots\ldots(36\,a)$$

If the members of such a system are each to consist of contact doublets, Θ_1 and Θ_2 will already be determined by (24). For the majority of such systems the problem of correction is evidently complex, and involves generally the selection of optical materials. Thermo-mechanical correction in the freer sense clearly demands systems of a somewhat greater complexity of optical construction, in which Θ_1 and Θ_2 may be given such values as will cause the above condition to be satisfied. Further consideration will be given below to such systems consisting of two separated doublets.

Reverting now to the general, chromatically uncorrected system, ($W \neq 0$), the effect of adding the formerly deleted W terms to (34) may be deduced from a consideration of the envelope of the family of lines in the (α_1', α_2') plane representing (33) with q as parameter. The equation for the envelope is most conveniently taken in the form:

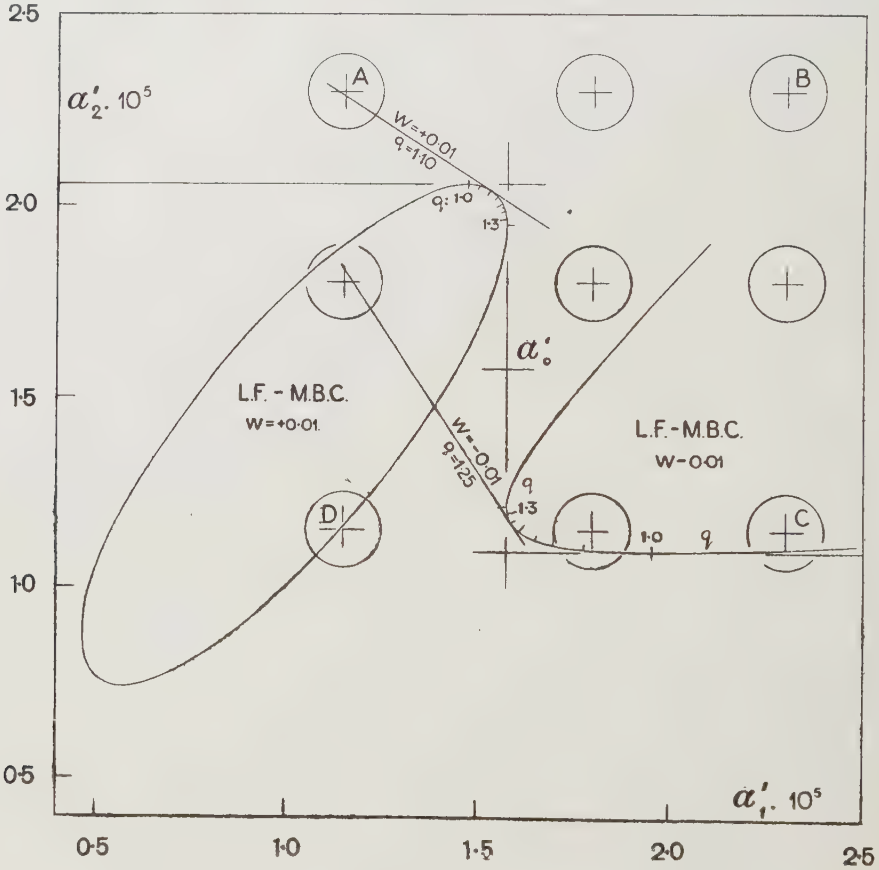

Figure 2.

$$w_1(\alpha_1' - \alpha_2') + w_2(\alpha_1' - \Theta_1) - (w_1 - W)(\alpha_1' - \Theta_2)$$
$$= 2\{w_2 W(\alpha_1' - \Theta_1)(\alpha_1' - \alpha_2')\}^{\frac{1}{2}}, \qquad \ldots\ldots(37)$$

where α_1' is regarded as the independent variable and α_2' is obtained after solving for $(\alpha_1' - \alpha_2')$. According as $w_2 W(w_2 - w_1)(1 - W)$ is positive or negative, this equation represents an ellipse or hyperbola, which is in either case tangential to the lines given by $\alpha_1 = \alpha_0'$ (constant) and $\alpha_2' = \alpha_0' + W(\Theta_2 - \Theta_1)/(w_2 - w_1)$ (constant), corresponding respectively to $q = w_1/w_2$ and $q = 1$. As was anticipated above, the envelope degenerates to a point (α_0', α_0') when $W = 0$. With numerically increasing W, the envelope develops out of and recedes from that point. Thus, (33) and (37) together determine in rectangular co-ordinates $(\alpha_1', \alpha_2', q)$ an elliptical or hyperbolic helix, according to the criterion above mentioned. The lines $q =$ constant by (33), tangential to the envelope, when considered as in three dimensions, will, of course, intersect the helix, which thus takes the place of the rectilinear axis of the simple conoid represented by (34). The q-lines thus generate in three dimensions a form of generalized double conoid. This surface may be represented in two dimensions in a manner similar to that employed above for (34) by a projection of the generator in various positions along its path; or, in condensed form, by the envelope (treated as the projection of the locus of the instantaneous centre of rotation of the generator) and graduated in a scale of q-values corresponding to the points of contact of a succession of tangential q-lines, as in figure 2. A q-line is there drawn tangentially for a specific q in each case as an example. Only the part of the conoidal surface corresponding to $1 \gtrless q \gtrless (w_1/w_2)$ is, as formerly, of interest in the present connection.

When the envelope corresponding to any given conditions is known, the q-line connecting pairs of values of α_1' and α_2' for any given q may immediately be drawn and alternative pairs read off, each capable of yielding thermo-mechanical correction in the given conditions.

§ 3.3. CALCULATION OF SPECIFIC SYSTEMS

On proceeding to apply these results numerically in specific instances, the first considerations must evidently be the limitations implicit in the optical conditions to be fulfilled and affecting the choice of optical materials and the general requirements as to the mechanical construction of the instrument in question. The latter will determine the extent to which the available, overall, effective expansion coefficients may be adjusted to accord with the results of calculation. Effective expansions intermediate or near to those of available materials may readily be obtained in practice by combining or opposing different materials, but the adoption of the more extreme values involving considerable over-compensation must generally be avoided out of consideration for the undesirable effect upon mechanical design. Other optical conditions, involving, for example, the chromatic residual, the Petzval sum, or power distributions to facilitate field corrections, will to a large extent predetermine the ultimate choice of glasses or other materials. Upon this choice depends the relevant values of W and Θ, from which an immediate indication may be derived of the region of expansion coefficient involved in the correction.

Applications to the doublet telescope objective have been referred to above. By table 3 it follows that thermo-mechanical correction is readily obtainable with some of the commoner combinations of glasses by the use of telescope mountings of the materials tabulated in table 2. Specific examples will now be given. These are, of course, subject to slight correction for finite systems. The doublet is here supposed in contact (although possibly not cementable).

The tube construction indicated is arbitrarily chosen to secure the tabulated α_2' value derived from table 3. Mean values have here been assumed for the expansion coefficients for steel and aluminium.

Glass combination	α_2'	Telescope tube construction for thermo-mechanical correction
H.C. (517.602) – D.F. (613.369)	2·17	89 % aluminium + 11 % steel
M.B.C. (573.576) – D.F. (613.369)	1·25	9 % aluminium + 91 % steel
M.B.C. (573.576) – E.D.F. (651.336)	1·45	26 % aluminium + 74 % steel

If the doublet has an air space, these data for tube material, while still applying to both α_1' and α_2', may possibly be adjusted to more convenient values by mutual adjustment between these values. It will be noted that doublet combinations with B.S.C. (513.637) glass are, generally speaking, unsatisfactory in this respect as they give inconveniently low values for α_0'.

Another practically important case is that of a compound system having one finite air space. As an example of such a combination we first consider here a separated two-lens system of a type suitable either for use on a small field unachromatized obliquely, or, as a basic form, subject to correction, for one-half of a quasi-symmetrical photographic objective. Having regard to the Petzval sum, P, and the α_0' value, we select from table 3 as materials the glasses L.F. (571.430) and M.B.C. (573.576), and, in order that the examples taken may be practically typical and comprehensive, we assume varying states of paraxial chromatic correction, viz., $W = -0{\cdot}01$, $0{\cdot}00$ and $+0{\cdot}01$ for the Fraunhofer spectrum range C to F and for unit Φ.

Assuming first $W = 0$, figure 1 relates graphically the expansion coefficients α_1' and α_2', securing θ-m. correction for this combination of materials in both the "crown-leading" and "flint-leading" forms, for a succession of q-values. The common point of intersection of the q-lines at (α_0', α_0') represents for either case the autogenous solution. The open lattice-like array of encircled points in this figure represents all possible combinations of immediately available α_1' and α_2' values, assuming the common materials: steel, brass and aluminium. The average dispersion and uncertainty of these values is exhibited with sufficient accuracy by circles of appropriate diameter. The most desirable solution will, of course, lie within the rectangle ABCD.

For certain definite values of q, the q-lines will pass through the circles above referred to, providing simple solutions in such case involving the use of only one kind of material in each mechanical part. If this condition is to be regarded as a determining factor in design, the q-values for which the condition holds may immediately be derived from the graph and the resulting systems evaluated, considered and compared for the purpose in view.

In other cases the expansion coefficients indicated graphically, being effective overall values, would be realizable by the use of appropriate combinations of different materials. As an example of such a more general case, let, by arbitrary assumption, q be given the value 1·18. The corresponding q-line is QQ', any point on which provides a solution in terms of α_1' and α_2'. We select alternative solutions at the points (i) and (ii). The resulting data for λ_D for the complete system, including the basic data derived by (26), are as follows:

Lenses: $\phi_1 = -7{\cdot}40$ (LF); $\phi_2 = +7{\cdot}12$ (M.B.C.). For glass types see table 1.
$d_1' = 0{\cdot}0244$; $q = 1{\cdot}18$.

Solution (i). $\alpha_1' = 2{\cdot}0$, $\alpha_2' = 1{\cdot}1$.

Resulting ΔB.F.L. $= +0{\cdot}0011\,\%$ per °c.

Component materials (measured axially) to give thermo-mechanical correction:

(*a*) Objective cell: 75 % brass + 25 % aluminium ; (*b*) camera body: iron or steel.

Solution (ii). $\alpha_1' = 1{\cdot}1$, $\alpha_2' = 2{\cdot}12$.

Resulting ΔB.F.L. $= +0{\cdot}0021\,\%$ per °c.

Component materials (measured axially) to give thermo-mechanical correction:

(*a*) Objective cell: iron or steel; (*b*) camera body: 45 % brass + 55 % aluminium.

The Petzval sum ($-0{\cdot}18$) is suitable for readjustment on assuming thicknesses or by adopting a slightly modified q-value.

When $W \neq 0$, the resulting envelope of the q-lines assumes finite form, as described above. Figure 2 represents the resulting envelopes for the two cases $w = +0{\cdot}01$, $-0{\cdot}01$ for the flint-leading form of the combination. The graduations on each envelope indicate the points of contact of the tangential q-lines for the q-values indicated, between $q = 1$ and $q = \nu_2/\nu_1$, the limits of the range of present interest. The line relating α_1' and α_2' for any given q may be drawn as desired, as tangent to the envelope at the point corresponding to q as given. Examples are shown for $W = +0{\cdot}01$, $q = 1{\cdot}10$, yielding, e.g., $\alpha_1 = 1{\cdot}1$ (steel), $\alpha_2' = 2{\cdot}3$ (aluminium); and for $W = 0{\cdot}01$, $q = 1{\cdot}25$, by which, e.g., $\alpha_1' = 1{\cdot}1$ (steel). $\alpha_2' = 1{\cdot}85$ (brass), these examples being chosen because of their simple solutions. The evaluation of the system and the mode of procedure in other cases will be clear from the foregoing.

Systems consisting of two separated members, each of which are contact doublets, include a number of types of lens combination of importance in the applications of photography. We shall here consider, by way of example, the θ-m. correction of a typical telephoto objective system of this form, assumed, in order to fix the ideas, to be schematically represented as to front and back members, by the basic data: $\phi_1 = +1{\cdot}90$, $\phi_2 = -1{\cdot}60$, $\alpha_1' = 0{\cdot}23$ for approximately unit Φ. It is not, of course, suggested that final data for such a system will follow forthwith from preliminary calculations of this kind; but important conclusions of a general character referring to the choice of glasses and the corresponding expansion corrections may very readily be derived therefrom. The essential relations are given above in (36). If the combination of glasses to be used in each member may be assumed, we may find by (36 *a*) the generalized mean, α_0', for the expansion coefficient α_1', α_2', and may effect further adjustment to bring these to correspond with those of available materials by (36). This method may readily be applied at sight to obtain approximate θ-m. correction in suitable cases of existing combinations.

If, as may happen, α_0' is given, at least approximately, as when the choice of materials consists in a relative adjustment of brass and aluminium for example, we may by (36 *a*), considering this as a linear relation between Θ_1 and Θ_2, correlate combinations of glasses suitable for use in front and back members respectively. By virtue of such a correlation it may be seen that, whatever the eventual α_0' value, some part of the total range of glass combinations will thus necessarily be excluded from use in the front member. If, for example, we assume $\alpha_0' \simeq 2{\cdot}0$, and allow Θ_2 to vary between the extremes $-1{\cdot}42$ and $3{\cdot}14$ of Θ'' in table 2, ignoring the H.C.–B.S.F. combination, the resulting corresponding total variation of Θ_1 is $1{\cdot}09$ to $2{\cdot}30$. (Such important glass types as the tabulated D.B.C. and B.S.C. are thereby excluded in such a case from the front member of the combination.) Correlated values, Θ_1 and Θ_2, will thus be found and various combinations securing θ-m. correction in the given conditions may be selected therefrom, an example from the tabulated glasses being M.B.C.–D.F. in the first member, followed by D.B.C.–D.F. in the second member; one may then proceed to consider combinations such as these from the point of view of general aberrational correction in the intended system.

§4. HIGHER-ORDER EFFECTS

When, for any given combination and set of conditions, the thermal variation, $d\xi/d\theta$, is reduced to zero, the θ-m. correction may be said to be stable. The outstanding thermal errors, which are then still theoretically possible for finite temperature ranges, may in practice be negligible. The effect of thicknesses upon the results of calculations similar to the examples given may perhaps be referred to here. This will generally not be entirely negligible, and may be ascertained from the foregoing theory. There are, however, other true higher-order effects which should here be mentioned. The first of these concerns the thermal variation of the chromatic aberrations in the case of primary aberrations and of first-order and second-order corrections. The seriousness of such effects will, of course, depend upon their magnitude considered relatively to other errors, but, judging from the small magnitude of the effect in some typical cases, such forms of thermal errors would rarely be of serious importance. The same remark applies to another higher effect, viz., the thermal displacement of the absorption band of light filters. As an example of this in the extreme case of a selenium glass reference may be made to Escher-Desrivières *et al.* (1940). This effect may, of course, acquire significance if the effective wavelength band of transmission is considerably reduced. It is not necessary more than to mention here the importance of thermal stability in the neighbourhood of thermopiles and other radiometric devices and in cases where delicate mechanical adjustments are involved. The thermal effect upon monochromatic aberrations may occasionally be of importance; but in such cases $\delta\epsilon$ will be of a magnitude rendering thermostatic control necessary. Consequently this effect is conveniently grouped with the heterogeneous effects.

§ 5. FUNDAMENTAL DATA

Various references are made in the literature to the temperature coefficients of refractive index and to the thermal expansions of optical materials, but few are of value as being sufficiently precise and reliable for the present purposes. It is, moreover, necessary, when considering optical glasses from this point of view, to be more than usually cautious in correlating the constants in question and in specifying the material to which they actually refer. In order that expansion and refractive properties may be related, other data, such as chemical composition, need to be taken into account (as has been done in some cases), and these are usually unknown for the glasses one wishes to use, although such data may be available in certain instances.

Also, in view of the fact that the expansion effect contributes to $dn/d\theta$, the same lack of immediate correlation must exist between this quantity and the (n, ν) datum. Thus, the properties n, ν, α and $dn/d\theta$ will, in effect, typify a glass in a much more restricted sense than does the number-pair (n, ν) alone, and it is only in this restricted sense that one may at present speak of a glass type in this connection. It appears to the writer a subject for reasonable conjecture that the triple number (n, ν, α) might specify such glass types sufficiently for such purposes. The conventional six-figured number representing (n, ν) would lend itself readily to extension of form for this purpose, and, simultaneously, to varying requirements and conditions of accuracy, if its parts were separated by a dot. Thus, well known glass types might in this form be specified as to (n, ν, α) to two significant figures in each part as 61.57.66 (cf. table 1, first line). When applying the previous results, the constants used should be those verified as pertaining to the particular glass types available. Such increased specificity does not exist, of course, with regard to the natural materials. There is, it may be mentioned, an extreme paucity of such correlated physical constants for optical glasses of other than Continental origin, and it may be permissible to express the hope that this hiatus in the available data may one day be filled. For the evaluation of thermo-optical effects over considerable temperature ranges, an extension of the fundamental data to include second-order temperature coefficients of expansion and refractive index would, of course, be necessary.

§ 6. NOTATION AND NOMENCLATURE: GENERAL OBSERVATIONS

The notation here employed is essentially that of Abbe, with slight changes and modifications where indicated and rendered necessary by the context.

Particular reference may here be made to minor departures in nomenclature here found desirable. The subject calls for clear notions in regard to conditions at contiguous surfaces. Optical parts are here referred to as *in contact* if the separation is zero or negligible as such, between the surfaces in question; and as *cementable* if, being in contact, the contacting surfaces are equal in curvature. In this and other allusions to joining by "cementing", no cementing properties are essentially implied other than the capacity to unite optically the surfaces in question. Glass types in the usual sense are designated here in the manner now current, by combining in one composite number the first three significant figures of $(n_D - 1)$ and of Abbe number, ν, e.g. 610.574, a dot being introduced here as separator, as above explained, as this renders possible a variable degree of discrimination between types and facilitates reference to broader classes, such as 61.57, and to sub-classes, such as 610.574.66 (see previous section). Coefficients of thermal expansion, α, and of thermal variation of elemental focal-length, ϑ, are expressed throughout in units of 10^{-5} per degree C.

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DISCUSSION

Dr. H. Spencer Jones. Mr. Perry has not dealt with conditions of non-uniform temperature, and has thereby side-tracked the effects of changes of temperature that are most troublesome to the astronomer. In order to illustrate these I will mention the experience we have had with the 26-inch refractor (focal length 26½ feet) at Greenwich. The instrument is rather large for the dome in which it is housed, and in consequence the dew-cap was of insufficient length to prevent dewing of the objective under certain atmospheric conditions. To prevent dewing, and the consequent loss of observations that it entailed, a wire was coiled spirally inside the dew-cap, not in close proximity to the objective, and heated by an electric current to warm the air in the dew-cap by a few degrees. This got rid of the trouble from dewing but was found to cause a marked deterioration in the quality of the images. Instead of being sharp and black, they became diffuse, and it was not possible to obtain sharp images by change of focus. The telescope was being used for the determination of stellar parallaxes, which demand a higher degree of precision than any other type of astronomical observation. It is important that the probable error of a single plate should not exceed $\pm 0''\cdot 025$. The probable errors of the plates with the diffuse images were appreciably greater, the residuals tending to be both large and erratic. The dewing trouble was in the end eliminated by lowering the whole telescope in its mounting by about two feet and providing a much longer dew-cap.

With this instrument the change of focus between summer and winter is within 0·5 mm., the compensation between the expansion of the steel tube with increase of temperature and the increase in focal length of the objective being almost perfect. But if the observer commences to make observations immediately after opening the dome, which has become warmed during the daytime, the tube cools more rapidly than the objective, with the result that the plates are inside the true focus. The observer is therefore required to open the dome at least one hour before commencing observations, in order to give the objective time to cool down to the temperature of the outside air.

Thus a considerable change of temperature is immaterial, provided objective and telescope have acquired a steady temperature, whereas a few degrees difference of temperature between the front and back faces of the objective result in a marked deterioration of the images, presumably by causing a change in curvature of the front surface.

Stratification of the air in the tube of the telescope also proved to be troublesome. To prevent this, a series of holes was drilled in the upper end of the tube, just behind the objective, and cool air was drawn into and through the tube by means of a fan fitted to its lower end. This also helped to ensure that front and back surfaces of the objective were exposed to air at the same temperature.

In astrometric work the astronomer is not normally concerned with the effects of change of scale with temperature. He uses a number of reference stars whose positions are known, and computes their co-ordinates on the plate, using an adopted scale value. The differences between the measured and computed co-ordinates are due to errors in scale and orientation of the plate, to the effects of refraction and to errors in the adopted positions of the reference stars. By using expressions of the type

$$\Delta x = ax + by + c,$$
$$\Delta y = dx + cy + f,$$

and solving by least squares, the errors of scale and orientation and the effects of the first-order terms in differential refraction and aberration are automatically allowed for, and the effects of errors in positions of the reference stars are reduced.

By computing and eliminating the differential refraction and aberration, the scale and orientation corrections can be derived, when desired, by using expressions of the type

$$\Delta x = ax + by + c,$$
$$\Delta y = -bx + ay + f.$$

This is sometimes preferable when the number of available reference stars is small or their positions are rather uncertain. Applied to the 13-inch astrographic refractor (focal length 13 feet 3 inches) at Greenwich, it has been found that there is no detectable effect of temperature on the scale ; this instrument has a steel tube. With a similar instrument at the Cape, by the same maker, there is, on the other hand, a very small, though perfectly definite, dependence of the scale upon the temperature.

When using reflecting telescopes, the effects of temperature changes can be much more serious. The effect on the focus of a gradient of temperature through the mirror as small as 0°·2 can be appreciably greater than the effect of a temperature change of 10° or 20° under steady conditions. With the 36-inch reflector at Greenwich the procedure has been adopted of refocusing at short intervals throughout the observations by means of the Foucault knife-edge test, which enables the focus to be checked with great accuracy in a matter of a few seconds.

In the observations with the floating telescope the scale cannot be controlled by the stars under observation, so that an accurate determination of the effect of temperature on scale is required. The objective is a Cooke triplet, with 6½ inches clear aperture and about 65 inches focal length. The tube is of brass. In this case, the proportional effect on scale (increase of focal length minus expansion of tube divided by focal length) amounts to $6{\cdot}3 \times 10^{-6}$ for 1° F. rise in temperature. For a rise in temperature of 50° F., the proportional increase in scale value (linear value of one second of arc) is 0·00031.

The Rev. W. REES WRIGHT. I would suggest that the main way in which thermal effects touch the astronomer is in the behaviour of such things as mirrors with changing temperature. I recall how a nine-inch unsilvered mirror I used for solar observations changed its focus as it heated up ; for a few minutes definition would be excellent, then it would go off until the mirror was unusable for a minute or two—then suddenly come good again, but with a change of focal length that was quite perceptible, and so on (the phenomenon is well known to all solar workers, but I fancy unrecorded for that very reason). Then again, the Mount Wilson people experienced very much trouble from the uneven expansion, and consequent loss of figure, of the coelostat mirrors in the Snow horizontal telescope, and again in the 60-foot tower telescope : I think that they have got over that difficulty of late years by employing Pyrex glass for all mirrors exposed to heating. Incidentally, I think that users of optical apparatus, who are not also opticians themselves, have very little idea of the extent to which a small amount of local temperature gradient can upset the figuring of an optical surface : it is a fairly common trick of amateur opticians, when showing the Foucault test to the uninitiated, to touch the mirror surface with the finger, or a small warmed rod ; although the heat is applied for a very brief time, and the expansion of the glass must be remarkably small, it produces a hump which, if it formed a zone on the disc, would cause the maker to return to the fine-grinding stage again (this, by the way, can be demonstrated very effectively with a concave lens surface of two or three feet radius).

Mr. J. HOME DICKSON. The thermal effects on long focus lenses are of considerable importance in air photography, where extremes of temperature are usual. Reports of bad definition with these lenses had been received and, as a consequence, a series o experiments was made to ascertain the effects of high and low temperatures on two [40-inch Telephoto] camera lenses of the type in general use. The experiments were made with as much care as possible in the time available, but immediate results were required, and it was not possible to carry out an exhaustive enquiry.

The temperature range covered was from $-30°$ C. to $+40°$ C., and it was found that the position of the best focus relative to the lens changed in a very decided manner, with the result that the camera, as a whole, could only be expected to remain in focus over a comparatively short range of temperature of about 20°. In addition, it was found that the glass showed strain, possibly caused by the unequal expansions of the glass and the metal mount, at about 35° C., and the definition in the image was very poor at both high and low temperatures, quite apart from the shift of the focus.

It was also noted that the back focal length and definition took a considerable time to return to normal after any changes had taken place, but that the lenses always returned eventually to their original condition, showing that no permanent harm had been done.

Dr. W. D. Wright. I remember hearing that a series of photographs taken by the late Professor Fowler during a solar eclipse expedition was spoiled because he overlooked the effect of change of focus due to the drop in temperature during the eclipse. It would be interesting to know what steps are usually taken by astronomers to overcome this trouble.

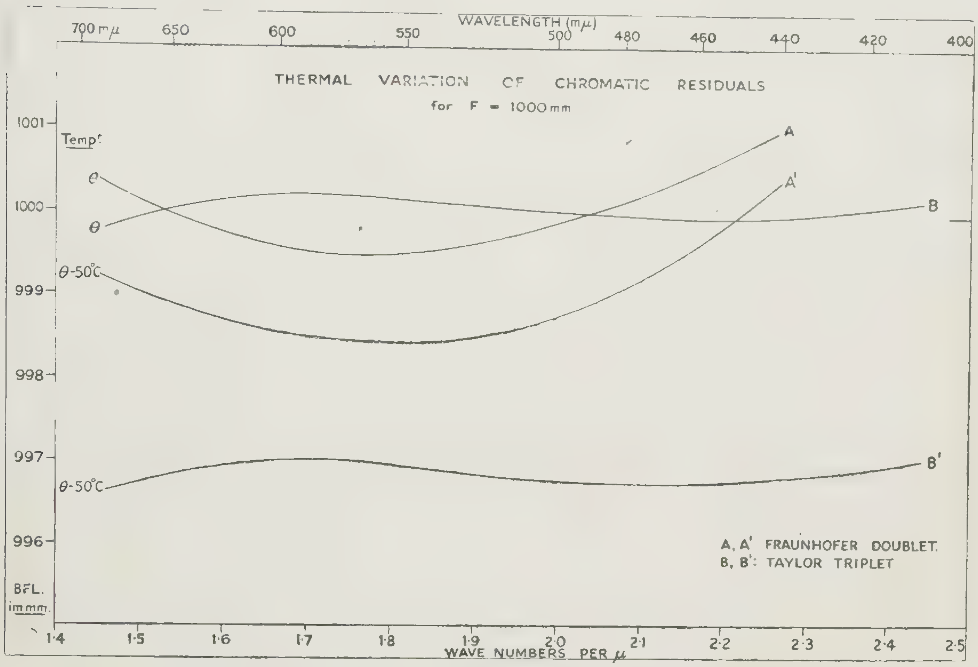
Mr. W. C. Hynd. I am surprised that no mention has been made of the effect of temperature change on the dispersion characteristics of glass and the consequent effect of this on the chromatic correction of lens systems. In measuring the dispersion of a crown glass on a critical-angle refractometer fitted with a dense flint prism a temperature change of 2° C. can cause a change in the measured C–F dispersion of 0·00001, a difference which computers in the past have stated to be significant in affecting the performance of lens systems. This change in chromatic correction with temperature is, moreover, an aberration, and cannot be corrected by a change of focus, as can the effects of the change in absolute index mentioned by earlier speakers. It seems possible that the temperature change of dispersion may account for the observation, reported by Mr. Dickson, that aerial camera lenses give very poor definition at high and low temperatures.

Author's reply. The point of view of the astronomer, presented by Dr. Spencer Jones, is of considerable interest to this subject, and represents a class of conditions differing from those of some other applications. In the case of astronomical instruments the effects of heterogeneous temperature conditions are the more troublesome mainly on account of the fact that effects of uniform temperature change, although very appreciable, frequently either may be removed by suitable adjustment of the instrument in the course of operation, or are compensated to a considerable degree, as I have shown for the case of refracting instruments, by the effects of the expansions of metal mountings. The optical effects of non-uniform temperature change depend very largely upon individual conditions, and obviously form a very important branch of the present subject. In the paper I have endeavoured to treat this generally from the point of view of optical performance, but more detailed consideration involving the assumption of specific instruments and observing conditions was not within the scope of the paper. Non-uniform thermal conditions are so dealt with in the first two sections.

On the subject of reflecting systems, referred to by the Rev. W. Rees Wright, it should be mentioned that the homogeneous thermal effects for such systems follow from the formulae given by assuming $n' = -n$. The interesting diagrams* shown by Mr. Dickson indicate, in addition to "homogeneous" effects, also what would seem to be defects such as may be introduced at cemented contacts. From the evidence it would seem that, of the methods of compensation referred to, the most appropriate, apart from the more radical question of re-design, would be increased circulation by between-lens ventilation. Sprung fittings would appear preferable in such cases to minimize mechanical strain.

* Not reproduced in this report of the Discussion.—Editor.

The thermal variation of chromatic aberration, referred to by Mr. Hynd, is certainly an important type of thermal effect, but judging by the results of calculations made upon some typical systems, a considerable measure of self-compensation generally takes place, owing to the combination of different materials, at least in the more usual cases of systems made of optical glasses. This has been found both with achromatic and apochromatic systems. The accompanying diagram shows the effect upon the residual chromatic aberration of the Fraunhofer doublet and of the triple apochromatic combination referred to in the course of the paper. The curves represent the residuals at normal temperature, θ, and at $(\theta-50)^\circ$ c.



These and all similar data should, of course, be interpreted differentially in relation to normal temperatures. Present conclusions of this nature are naturally limited in this respect to information based upon the available data, and it may later be found that the thermal variation of dispersion for various glasses does not in some combinations result in such a favourable degree of compensation as is here indicated, whence is apparent the need for increased information in this direction. Data at present available enable one for any thermal variation to calculate strictly only the differential coefficient applicable under normal conditions. What still remain in doubt are the higher-order thermal effects, important in the case of large temperature changes.

THE LIMIT OF VISUAL RESOLUTION

By E. W. H. SELWYN

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ABSTRACT. The diffraction image of a long line source of light at the focus of a lens with a circular aperture has been calculated numerically. These figures are used to find what intensity drop between two bright lines is at the threshold of visibility, by making observations on the separation of two line sources at which a dark line is visible between their images. After means had been taken to avoid a well-known contrast effect which produces a false dark line, it was found that, on the average, a 3 per cent drop of intensity is required for threshold resolution.

§1. INTRODUCTION

The classical definition of resolving power, due to Rayleigh (1888), requires that for two equally bright point sources to be seen separated in an optical instrument, their distance apart must be at least that at which the central maximum of one diffraction image falls on the minimum of the first dark ring of the other diffraction image. The ratio of maximum brightness to the brightness at a position intermediate between the two central maxima is then about 4 : 3. There is considerable evidence that resolution is possible at closer distances than this rule would indicate. Thus, Dawes (1867) gave the general formula 4·56/*D* for the angular resolving power, in seconds, of a telescope, the objective of which is *D* inches in diameter. The separation between the two point sources according to this formula is about 0·8 of that suggested by Rayleigh's. Martin and Johnson (1931) and Conrady (1929) also mention that resolution is possible of finer detail than is suggested by Rayleigh's definition. It is not a matter of great importance that any conventional definition of resolving power should give a somewhat inaccurate value, for the definition of the limit of resolution is bound to be arbitrary in some degree; but in connection with the investigation of the resolution in camera negatives, it seemed desirable to examine this question of the limit of resolution a little further. It is sometimes stated that the resolution of two point sources is dependent upon a fall in brightness between the centres of the two images. The question at issue was what the amount of this fall had to be for resolution to be apparent between the images of a number of equidistant long parallel line sources. The resolution of such sources can only depend upon the nature of the diffraction images and the drop of brightness between the centres of the superimposed images, and is, therefore, a simpler question to consider than that of two point sources, where the shape of the apparent image also comes into play. Supposing, for instance, that conditions were good enough for an astronomer to recognize the Airy disc of some object as being a little elliptical, then the object might well be recorded as a double star, or possibly a small nebula, although it could by no means be regarded as resolved. There would, in this case, be no drop in intensity in the middle of the pattern. A spectroscopist, on the other hand, is entirely dependent upon observing a drop in intensity between two lines in order to recognize the diffraction image as that of two lines, for only in exceptional circumstances can he have any *a priori* knowledge of the actual width of the lines, which is the only other guide available to him. Ditchburn (1930) has suggested that, if the observer has a knowledge of the width of a single line, inhomogeneity due to the existence of two close lines can be detected at the separation at which there is just no drop in intensity in the middle of the image.

§ 2. DIFFRACTION IMAGE OF STRAIGHT LINE SOURCE

The image given by a lens with a circular aperture of a straight line source may be considered as built up of an infinite number of Airy diffraction patterns of a point source arranged indefinitely close together along a line. Numerical results for the intensity distribution in such an image, obtained apparently by numerical integration, were given by André (1876), while series for the calculation of the distribution were given later by Rayleigh (1888). This distribution has now been recalculated with results much more accurate than those given by André. The calculations were made in the first place by numerical integration of the figures already available for the intensity distribution in the image of a point source. The figures so obtained were then checked by calculating values from Rayleigh's series,

$$1-\frac{4z^2}{3\cdot 5}+\frac{4^2z^4}{3\cdot 5^2 . 7}-\frac{4^3z^6}{3\cdot 5^2 . 7^2 . 9}+\dots,$$

where $z=2\pi Rr/f\lambda$, R being the distance of the point for which the intensity is required from the central line of the image, r the semi-diameter of the aperture of the lens, f the distance of the image from the rear nodal point, and λ the wavelength. The first ten values, and others at intervals throughout table 1, were

Table 1. Intensity in diffraction image of line source

z	Intensity	z	Intensity	z	Intensity
0·0	1·0000	3·2	0·0463	6·2	0·0153
0·2	0·9894	3·4	0·0349	6·4	0·0127
0·4	0·9581	3·6	0·0323	6·6	0·0114
0·6	0·9097	3·8	0·0329	6·8	0·0110
0·8	0·8413	4·0	0·0359	7·0	0·0114
1·0	0·7619	4·2	0·0395	7·2	0·0123
1·2	0·6737	4·4	0·0424	7·4	0·0133
1·4	0·5809	4·6	0·0440	7·6	0·0143
1·6	0·4877	4·8	0·0439	7·8	0·0148
1·8	0·3978	5·0	0·0420	8·0	0·0150
2·0	0·3151	5·2	0·0384	8·2	0·0146
2·2	0·2416	5·4	0·0339	8·4	0·0136
2·4	0·1791	5·6	0·0287	8·6	0·0124
2·6	0·1286	5·8	0·0236	8·8	0·0109
2·8	0·0900	6·0	0·0190	9·0	0·0094
3·0	0·0626				

thus confirmed. In only one case was the discrepancy as great as 2 in the last figure. The figures given are therefore believed to be reliable to within one or two units in the fourth decimal place. The two methods of calculation are, in a sense, complementary, for numerical integration is much less laborious than calculation from the series for the higher z-values and *vice versa* for the lower values. Rayleigh also gives a series for calculating the intensity at large values of z.

§ 3. OBSERVATIONS ON RESOLUTION

A test-object, consisting of a series of well separated pairs of transparent parallel straight lines in an opaque background, was made by photographing a carefully made drawing on a "Maximum Resolution" plate. The ratio of the width of the lines to the separation was about 1 : 20. This plate was illuminated from behind by a sodium lamp and observed by means of a small telescope from a spectrometer. A number of small stops was provided for the objective of the telescope, and observations were made of the distance of the test-object at which the image of a given pair of lines appeared with certainty to have a drop in intensity in the middle of the image. The angular separation of the test-objects measured varied from $\frac{1}{4}$ to 1 minute, and the diameter of the aperture over the telescope objective from 2 to 8 mm., the telescope magnification remaining approximately constant at 14×. The value thus found for z', z' being equal to $2\pi R'r/f\lambda$, in which R' is the observed separation between the geometrical images of the lines, was 2·80. A considerable number of observations was made, and this figure is reliable. But in actual fact there is no drop in intensity at this value of z'; the curve of intensity is flat-topped. An image with a flat-topped or slightly humped intensity curve was therefore provided, in a different way, by a relatively wide slit of uniform width, observed by means of the telescope fitted with an aperture small enough to give a smooth drop in intensity at the edges of the image. This image was found also to display a dark central band. Resolution of a pair of lines can therefore be quite spurious. The effect is well known in other forms as a visual illusion. It is due to some effect of contrast between the bright image and the dark background. The same effect was described by Sparrow (1916), who also examined the effect of unequal brightness of the lines on the separation at which apparent resolution occurred. The experiments showed that apparent resolution occurs when there is no dip in the intensity curve for two equally bright lines, or when the curve between two lines of different brightness shows no appreciable deviation from straightness.

A "multiplet" of four equidistant parallel transparent lines was then made in a similar manner to the first test-object. In this case the ratio of the width of the lines to the separation was 1 : 30. The reason for adding the two outer lines was to shield the inside pair from the above disturbing contrast effect, so that any darkening observed between the two inside lines would be real. Observers were then asked to say whether they could see any dark line between the two middle bright lines of the pattern at various distances of the test-object from the telescope. Only the one test-object and an aperture over the telescope objective of 4 mm. diameter were used in these observations. The previous measurements had revealed no systematic differences for the different conditions of observation tried. Results were obtained by thirty observers, some skilled in the use of optical instruments, and in particular in photographic resolution measurements, and others not so skilled, but not without useful experience of optical apparatus. Analysis of the observations, which are shown in table 2, shows that they follow approximately a Gaussian distribution with a mean z' value of 3·18 and a standard deviation of 0·05. This corresponds with an intensity drop in the centre of a four-line pattern of 3·2 per cent and a standard deviation of 1·7 per cent.

The observers were not aware of the direction or amount of movement of the test-object between any two observations. Although, therefore, no opportunity was given for spurious improvement on an observer's earlier attempts at resolution, it was noticeable that an observer's later observations often gave lower z' values than his earlier ones. A few observers made observations (namely those for which z' values of 3·08 were obtained) which suggest that they claimed to be able to resolve lines so close that there was no drop in intensity between them. It is difficult to rule out imagination in these cases.

It seems likely that there are two classes of observer (with, of course, intermediate types), namely, those who require the darkening between the two lines to be definite, and recognizable without forced attention, and those who are prepared to go to extremes in examining the image to determine whether there is any resolution. Observers who were judged to belong to the first group were quite definite in their opinions whether the lines were or were not resolved, and in general could repeat their individual observations of z' values to within about ±0·02. Those who were considered to belong to the second group, a much smaller number, were less definite, and their repetition error was two or three times that of the first group. In addition to such differences in opinion of what constitutes resolution, there are also undoubtedly differences in contrast sensitivity between observers. The value of 3·2 per cent arrived at by these experiments is of the same order of magnitude as other estimates. Leaving out of consideration the measurements by Sparrow, which apply only to the spurious resolution already described, the remaining direct estimates are those by Hartridge (1922) and Lapicque (1938), both of whom calculated the intensity distribution in images on the retina and compared the results with data on visual acuity, and Reese (1939), who examined the visibility through a telescope of thin wires against a bright background. Hartridge's estimates approximated to 10 per cent, while Lapicque arrived at a value of 2·7 per cent. These figures apply to objects at the limit of resolution of the eye, when the pupil of the eye has dimensions of 2 or 3 mm., whereas in the present experiments the exit pupil (and therefore the effective pupil of the eye) was only 0·3 mm. in diameter.

Table 2

z'	3·08	3·095	3·13	3·145	3·16	3·18	3·20	3·22	3·24	3·26	3·32
No. of observers	2	2	1	5	1	6	5	5	1	1	1

Exactly the same image on the retina as is obtained by observations through the telescope would be obtained by observation of a similar test-object of such a size that it was just at the limit of resolution, when an aperture 0·3 mm. in diameter was held in front of the eye. At this pupil aperture, the lines will require to be separated by approximately 5 minutes to be resolved. Further, a close approximation to the same image on the retina would be obtained with the pupil of the eye at a normal opening, provided that the test-object consisted of relatively broad bands separated by intervals of approximately the same width, but with only a small difference of brightness between the bands and the spaces between them. Since the sharpness of the image produced by the eye is some five times greater at its normal aperture than at 0·3 mm., the degradation of this last image by the eye is relatively unimportant, and the contrast difference in the test-object approximates to the contrast-difference in the image on the retina. From the data of Cobb and Moss (1928) it appears that resolution occurs for test-objects consisting of two broad dark bands separated by an interval of the same width as a band and with a separation between centres of the dark bands of 5 minutes, when the contrast figure is 4·6 per cent at 100 ml. or 6·8 per cent at 20 ml. Since there is some degradation of contrast by the eye, these figures are necessarily upper estimates for the contrast of the image on the retina. Moreover, the observer was allowed 0·17 second only during which he could see the test-object, a condition which would tend to require higher contrast for resolution than if the observer were allowed to take his own time. On the other hand, since the test-object consisted of a pair of lines only, the contrast effect previously mentioned may have led to lower values than when that contrast effect is eliminated. On the whole, however, the agreement with this indirect estimate is not unsatisfactory.

Reese found that a thin black wire viewed against a bright background through a telescope disappeared when its diffraction image was so broad that the brightness in the middle of the image was about 6 per cent below the brightness of the surround. This figure is not subject to any appreciable correction for the contrast effect, because there were no considerable changes of brightness within the field of view. Whether the agreement with the present estimate of 3 per cent is really satisfactory for experiments so nearly similar is perhaps doubtful, but in this respect it should be noted that the brightness distribution in the field of view was different in the two cases, and that in any case the variation among observers is considerable.

Another rough estimate is possible from considerations of the resolution of two lines and the visibility of a thin line. Two thin bright lines are normally resolved at a separation of about 60 seconds, which means that the width of the image of a single line is of the order of 60 seconds also. The image of a thin black line of width x seconds with a brightness zero against a background of brightness unity will then be a band of width 60 seconds and brightness $1-x/60$, because, in a sense the amount of black in a width x has been expanded to a width 60. The actual thickness of a black line which is just visible is three or four seconds. It must be concluded, therefore, that the wire is just visible when the difference of brightness of its image and the background on the retina is of the order of 5 or 6 per cent.

Finally, it follows from the comparison of the apparent resolution of the two-line and four-line test-objects that the apparent darkening due to the contrast effect is equivalent to a real reduction of intensity of about 3 per cent.

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RECENT OPTICAL MATERIALS AND THEIR POSSIBLE APPLICATIONS

By B. K. JOHNSON, D.I.C.,
Technical Optics Department,
Imperial College of Science and Technology

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§ 1. INTRODUCTION

The position of optical science today is limited to a large extent by the optical materials available. Within the last ten years quite a number of new materials suitable for optical work have appeared, and when full use of them has been made it is quite possible that advances both in optics and physics may result. In this paper an attempt has been made to describe the optical properties of some of these materials and to suggest some of their possible applications. Whilst these remarks may chiefly be of interest to optical designers, they may also be found helpful to spectroscopists and physicists in general.

During the past forty years there has been a large variety of optical glasses which the designer has utilized for producing satisfactory lens-systems practically free from the undesired aberrations. But if there were materials which had a greater range of refractive index and dispersion, this would still further assist his work. As far as the spectroscopist is concerned, the work in the invisible parts of the spectrum (whether infra-red or ultra-violet) has hitherto been carried out by the use of natural optical crystals which are limited both in size and quality, but with the introduction of the new and extremely large synthetic crystals, spectroscopic work may be greatly extended.

§ 2. GLASSES (OPTICAL)

At the present moment, in most glass catalogues, the range of refractive indices available is from 1·47 to 1·81, with V numbers varying from 70 to 25. Unfortunately, however, it is not possible to choose a glass having any required V number with a given refractive index. Also, the partial dispersions are such that with glasses alone it is not possible to make a lens combination entirely free from colour, but if one of the elements of the lens system is made from a material containing a mineral salt (e.g. calcium or lithium fluoride) the colour-correction may be brought very near indeed to the ideal.

For optical designing it would be advantageous to have glasses with:

(*a*) a high refractive index and low dispersion;

(*b*) even higher refractive index and considerable dispersion.

If such glasses were available, it would be possible (by suitable choice of dispersion and refractive index) to make an apochromatic telescope objective with only two components. Similarly, if we had two glasses with almost the same refractive index, but with widely different dispersions, or with the same refractive index and dispersion but with different partial dispersions, then the secondary spectrum could be corrected more readily.

Table 1

Wave-length (A.)	Percentage transmission (10 mm. thick)					
	Uviol	Vita	Corex	Pyrex	Hard crown	Dense flint
3303	86	81	87	18	65	10
3133	65	72	85	3	10	—
3081	35	56	78	—	—	—
2837	—	11	59	—	—	—
2749	—	—	12	—	—	—
2446	—	—	—	—	—	—

Some new glasses which have been recently produced are:

(1) Phosphate glasses with refractive index of approximately 1·52 and V value of 70.

(2) Zinc borate glasses with refractive indices ranging from 1·63 to 1·65 and V numbers 50 to 55.

(3) A range of very high index glasses containing combinations of zirconium, titanium, tungsten, thorium, lithium and boron. These range from 1·71 up to 1·97, with V numbers from 54 to 19.

There has been a tendency towards rendering glasses transparent in the ultra-violet region of the spectrum. This has been accomplished to some extent, as illustrated in table 1. Corex glass has the best transmission of those quoted, but it is not particularly homogeneous, having the appearance of a rather high silica content. Hertzrucken (1936) claims that a glass containing beryllium fluoride has a marked transmission in the ultra-violet and is at the same time homogeneous. No progress appears possible in rendering glass transparent in the infra-red region, where a cut-off still remains at about 20,000 A.

§ 3. PLASTICS

The introduction of synthetic resins during the last few years has received considerable attention in various branches of industry, particularly in the electrical world, where the dielectric properties of these materials are valuable. Some of these resins, however, have been used for certain optical purposes, though not very much appears to be known of their optical properties. There are some twenty-five named varieties of these so-called plastics, the majority of which are either opaque or translucent. Six of them, however, are transparent and appear to offer possibilities for optical work; their commercial names are as follows: Perspex or Plexiglass, Bexoid, Celastoid, Distrene, Polystyrene, Xylonite. A considerable amount of work has been carried out by W. G. Wearmouth and H. Lowery in measuring the optical constants of certain of these materials, but this work is as yet unpublished. The writer has also carried out similar measurements independently with a view to determining the most suitable combination for producing achromatized lenses.

Table 2

Material	N_D	Mean dispersion C–F	V	Partial dispersions	
				C–D	D–F
Perspex	1·4881	0·0082	59·5	0·0025	0·0057
Polystyrene	1·5927	0·0193	30·7	0·0054	0·0139
Hard crown	1·5204	0·0087	59·9	0·0026	0·0061
Dense flint	1·6182	0·0170	36·4	0·0048	0·0122

One advantage in the use of this material is that it can be accurately moulded in steel dies so that the specimen comes out with finished and polished surfaces, thus saving the grinding and polishing processes usually employed in glass work. (The temperature at which the moulding takes place is between 280° and 330° F. and the pressure is about 2000 lb. per square inch.) It is claimed that surfaces of optical quality can be obtained by this process and that a finished radius will fit the glass test-plate to within two or three fringes. The homogeneity of the materials is stated to be good enough to comply with most optical requirements, and the refractive indices range from 1·49 to 1·66 for D light. The specific gravity is of the order of 1·0, whereas that of glass is approximately 3·0. Unfortunately most of these resins are relatively soft (Brinell Nos. 20 to 25), and are, therefore, liable to scratch rather readily; on the other hand, they are not liable to fracture, if dropped for example. A possible advantage is that if required they may be dyed to almost any colour.

At the present moment the writer is of the opinion that the two most hopeful varieties of these resins for achromatizing purposes are methyl methacrylate (Perspex) and trolitul (Polystyrene). Measurements of the optical constants of a sample specimen of each of these materials were made, and these are shown in table 2 together with the values for an ordinary crown and flint glass for comparison.

Table 3

Type of lens	Perspex combined with Polystyrene	Hard crown combined with dense flint glass
For an equi-convex front lens	$r_1=+\ 4{\cdot}728''$ $r_2=-\ 4{\cdot}728''$ $r_3=-31{\cdot}74''$	$r_1=+\ 4{\cdot}090''$ $r_2=-\ 4{\cdot}090''$ $r_3=+166{\cdot}67''$
For a "flat" last surface	$r_1=+4{\cdot}115''$ $r_2=-5{\cdot}555''$ $r_3=\ \infty$	$r_1=+4{\cdot}184''$ $r_2=-4{\cdot}000''$ $r_3=\infty$

The figures given suggest the possibility of producing an achromatic lens employing these two materials, and if some calculations are made in order to determine what radii would be necessary to fulfil the condition of achromatism for, say, a telescope objective of 10-inch focal length, we find the results given in table 3. Thus it will be seen that there are distinct possibilities in the use of synthetic resins for producing achromatized lens systems; but whether the lack of homogeneity, the softness of these materials, or other causes will prevent them from being generally used for optical purposes remains to be seen. The transmission properties of Perspex and Polystyrene in the ultra-violet are almost identical and are shown in table 4. In the infra-red region they absorb strongly at about $2{\cdot}0\,\mu$ and then commence to transmit again at about $20\,\mu$.

Table 4. Perspex and Polystyrene

λ (A.)	Percentage 10 mm. thick	Transmission 2·5 mm. thick
3340	32	75
3081	6	51
2980	—	25
2921	—	12
2880	—	5

§ 4. FUSED QUARTZ

Transparent fused silica has somewhat limited applications in optical work, but it can be distinctly useful in certain cases. Its chief advantage over the crystalline variety is that it is not bi-refracting. This is an essential point when a lens system is to be made from this material, where rays pass through the lenses at large angles of incidence (e.g. an ultra-violet microscope objective). The range of transmission (0·20 μ to 4·0 μ) is similar to crystalline quartz, but it is a substance which is difficult to make homogeneous. This will be readily understood when it is realized that the natural quartz crystals must be raised to a temperature of 1700° c., where as a liquid it must be stirred; the maintenance of this temperature and the time of fusion are points of considerable importance in the process. The mass has then to be cooled carefully without crystallization and annealed. Pieces as large as one and a half inches in diameter and half an inch thick (produced by the Thermal Syndicate Co., Ltd.) have proved to be quite homogeneous enough for optical purposes. The refractive index and dispersion of fused quartz make it very suitable for combining with lithium fluoride in order to produce achromatized lens systems for use in the visible and ultra-violet parts of the spectrum. Such lenses, both in microscope and telescope form, have been produced with success (Johnson, 1939 and 1941). The refractive indices for various wave-lengths are to be found in any standard reference on this subject, such as the *Dictionary of Applied Physics*, vol. iv, or Baly's *Spectroscopy*, and, therefore, it is not necessary to quote them here.

§ 5. THE ALKALI HALIDES

Within comparatively recent years efforts have been made to grow various optical crystals with a view to duplicating or augmenting natural crystals. In order to work in the ultra-violet or infra-red parts of the spectrum it is necessary to have such crystals for the optical components, but natural crystals limit the size of the latter owing to lack of homogeneity in anything but small pieces. With the coming of a new technique for growing a variety of synthetic crystals of really large dimensions (e.g. 10 inches in diameter), the possibilities of increased light-gathering power and resolving power of spectroscopic instruments must be considerable.

In the early stages of this work (1925 to 1930) several investigators (Gyulai, 1927; Ramsperger and Melvin, 1927; Pohl, 1932) produced artificial crystals of some of the halides, such as lithium fluoride, sodium chloride, and potassium bromide, but only on a small scale; Bridgman (1925) grew such crystals of about 4 cm. in diameter. Present-day commercial methods as used at the Harshaw Chemical Co., Cleveland, U.S.A., are based on a technique developed by Stockbarger (1936), and enable crystals to be grown which are as large as 10 in. in diameter and some 25 lb. in weight. The Bridgman method of using one furnace only was modified by employing two annular tube furnaces separated by a close-fitting metal diaphragm (see figure 1). The upper furnace is maintained at a temperature above the melting point of the salt (but not high enough to cause excessive vaporization), whilst the lower furnace is below the melting point, so that when the platinum crucible containing the molten salt is lowered through the diaphragm, crystallization takes place in the region of high gradient between the two furnaces. The method thus constitutes a so-called continuous crystallization process, with the rate determined by the speed at which the crucible is lowered. This latter is, of course, extremely slow, and it may take up to ten days to grow a crystal; during this entire period all mechanical operations, such as the lowering of the crucible, temperature of the furnaces, etc., must be carefully controlled. After crystallization is complete the temperature is again brought up to within 50° c. of the melting point of the salt and the crystal is annealed for seven or eight days. Figure 2 shows portions of various crystals grown by the method described, including one complete crystal of lithium fluoride 7 in. in diameter.

In the earlier stages of this work, crystals were annealed *in* their crucibles, and when cold the adhering platinum had to be carefully ripped off (see figure 3). Later, however, a method was developed (Stockbarger and Cartwright, 1939) whereby the crystal is melted out of its crucible. In this method the crystal and crucible are transferred to a special furnace and placed in an inverted position. The furnace temperature is then held slightly below the melting point of the salt for several hours, when shortly the crystal becomes released from the walls of the crucible; the crystal is then annealed as before. By this melting-out process the platinum crucible is saved and, after re-shaping, is again put into service (see figure 4). The crucibles are constructed of purest grade platinum, and have walls between 0·003 and 0·006 in. thick.

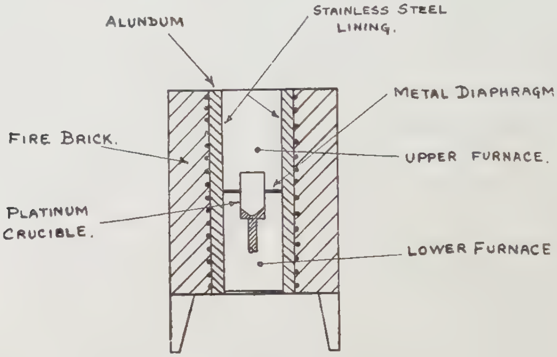

Figure 1. Stockbarger furnace.

§6. LITHIUM FLUORIDE

The crystals now grown of this material make it an effective substitute for fluorite (CaF_2). The optical constants (see table 5), the transmission range, and the absorption coefficients are all closely similar to those of fluorite, but it has the additional advantage that it can be obtained in much larger pieces, and further, it is more homogeneous. Thus it is obvious that this material is valuable for spectrographic work in the ultra-violet, and may be used also in the vacuum region down to about 1200 A. (Johnson, 1938). Measurements on the refractive indices and transmission at various wave-lengths have been determined by Schneider (1934) and Hohls (1937), and figure 5 shows these figures plotted. Lithium fluoride is particularly suitable for combining with fused quartz when achromatized lenses are required in the ultra-violet or even in the visible spectrum. Johnson (1939) has shown how ultra-violet microscopy may be greatly simplified by employing an almost apochromatic lens system utilizing these two materials. He has also shown (1941) that a telescope objective designed with LiF and

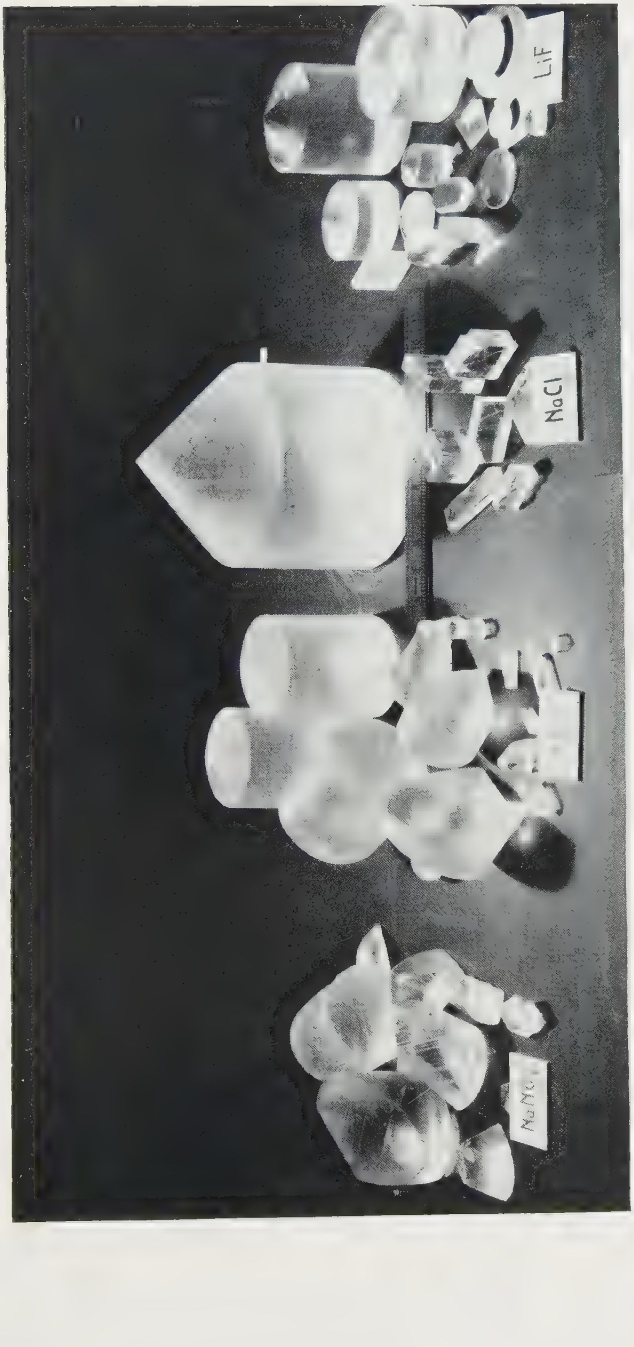

Figure 2. A group of crystals grown by methods described in the text.

(Courtesy of Harshaw Chemical Co.)

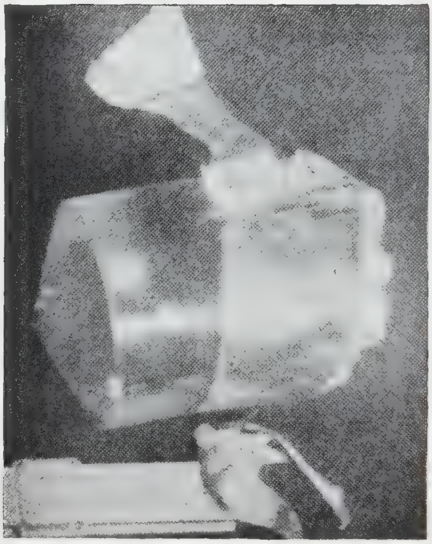

Figure 3. Crucible removed by peeling off.

(*Courtesy of Industrial & Engineering Chemistry.*)

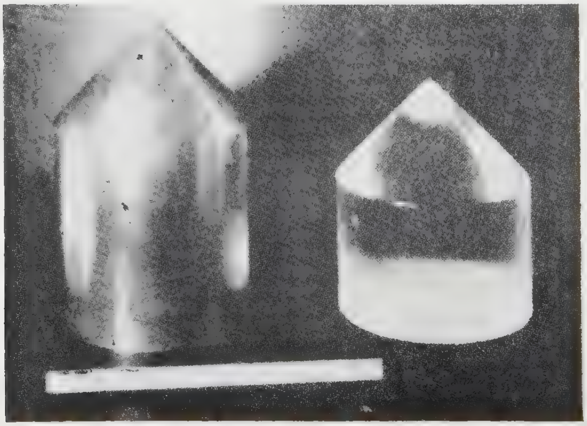

Figure 4. Crucible removed undamaged.

(*Courtesy of Industrial & Engineering Chemistry.*)

Table 5

Material	N_D	Mean dispersion C–F	V	Partial dispersions		Limit of transmission (μ) for 10 mm. thick
				C–D	D–F	
Fused quartz	1·4587	0·0067	68·5	0·0020	0·0047	0·19 to 3·5
Lithium fluoride	1·3922	0·0037	106·0	0·0016	0·0021	0·12 to 7
Fluorite (natural CaF_2)	1·4338	0·0044	98·6	0·0013	0·0031	0·13 to 8·5
Potassium bromide	1·5590	0·0165	33·9	0·0046	0·0119	0·21 to 28
Potassium chloride	1·4901	0·0111	44·1	0·0031	0·0080	0·20 to 21
Potassium iodide	1·6655	0·0284	23·4	0·0086	0·0198	0·20 to 35
Fused magnesium oxide	1·7378	0·0138	53·5	0·0041	0·0097	0·23 to 5
Rocksalt (natural)	1·5443	0·0127	42·8	0·0036	0·0091	0·20 to 15
Sodium chloride (synthetic crystal)	1·5498	0·0156	35·2	0·0055	0·0101	0·20 to 15
Perspex or Plexiglas	1·4881	0·0082	59·5	0·0025	0·0057	0·34 to 2·0
Polystyrene	1·5927	0·0193	30·7	0·0054	0·0139	0·34 to 2·0

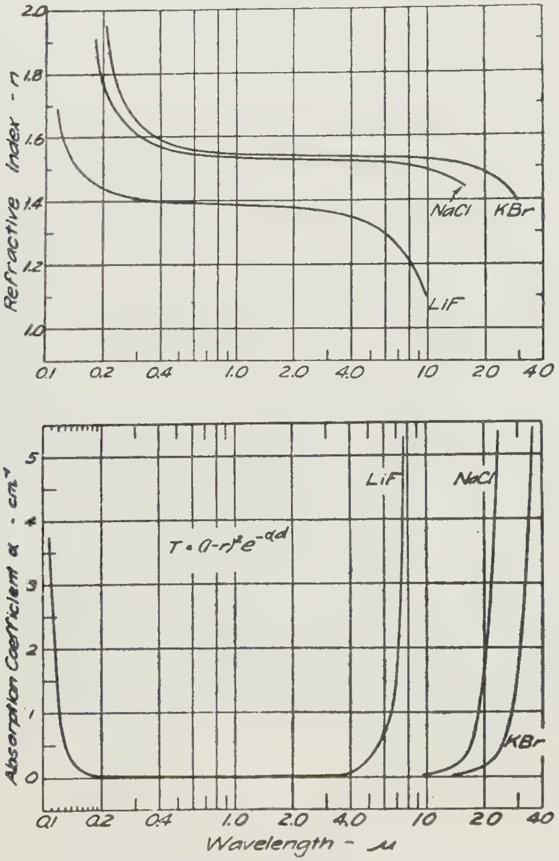

Figure 5. Refractive index and transmission at various wave-lengths for LiF, NaCl and KBr.

SiO_2 as the components has extremely small residual amounts of chromatic aberration within the range 6000 to 2500 A.

The optical working of lithium fluoride presents no unusual difficulties. For cutting the material into suitable blanks, a power hack-saw may be used, fitted with a copper blade and freely supplied with emery and water as an abrasive. The ordinary fine grades of abrasive should be used for the grinding processes, followed by a polishing medium such as tin oxide or magnesium oxide; rouge is rather too hard. Too rapid polishing tends to pull out fragments of the crystal and leave pits in the surface.

§ 7. POTASSIUM BROMIDE, SODIUM CHLORIDE, AND POTASSIUM IODIDE

Workers in the infra-red region of the spectrum will in all probability be interested in substitutes for natural rocksalt, although artificially grown crystals of this material now reach dimensions which were unobtainable in natural crystals of NaCl. Figure 6 shows a synthetic rocksalt 60° prism blank with prism faces of 8 in. by 5 in.

Potassium bromide has optical properties similar to rocksalt but, having a high molecular weight, it transmits further into the infra-red. Whereas the effective limit for NaCl is about 15 μ, that of KBr is 28 μ. The refractive indices of this last-named material in the region 0·21 μ to 0·6 μ have been determined by Melvin (1931), and between 0·8 μ and 28 μ by Gundelach (1930), and the absorption coefficients by Mentzel (1934). These values are plotted in figure 5. Potassium iodide can be grown under similar conditions to potassium bromide, and will transmit up to nearly 40 μ. Whilst the optical constants of KI have been measured for the visible spectrum (see table 5), data for the infra-red are not yet available. It does appear, however, from these values that an achromatized lens system for the infra-red region might be quite possible utilizing KBr and KI as components.

§ 8. SODIUM NITRATE

The growing of large crystals from the molten salt of $NaNO_3$ has been stimulated by the fact that these crystals are bi-refracting and resemble calcite to some extent in optical properties, and, therefore, may serve as a valuable substitute for the latter. The Harshaw Chemical Co. state that $NaNO_3$ crystals

Table 6

λ (A.)	Sodium nitrate		Calcite	
	Ordinary ray	Extraordinary ray	Ordinary ray	Extraordinary ray
5893	1·5848	1·3360	1·6584	1·4864
5011	1·5968	1·3370	1·6660	1·4899
4348	1·6126	1·3400	1·6755	1·4943

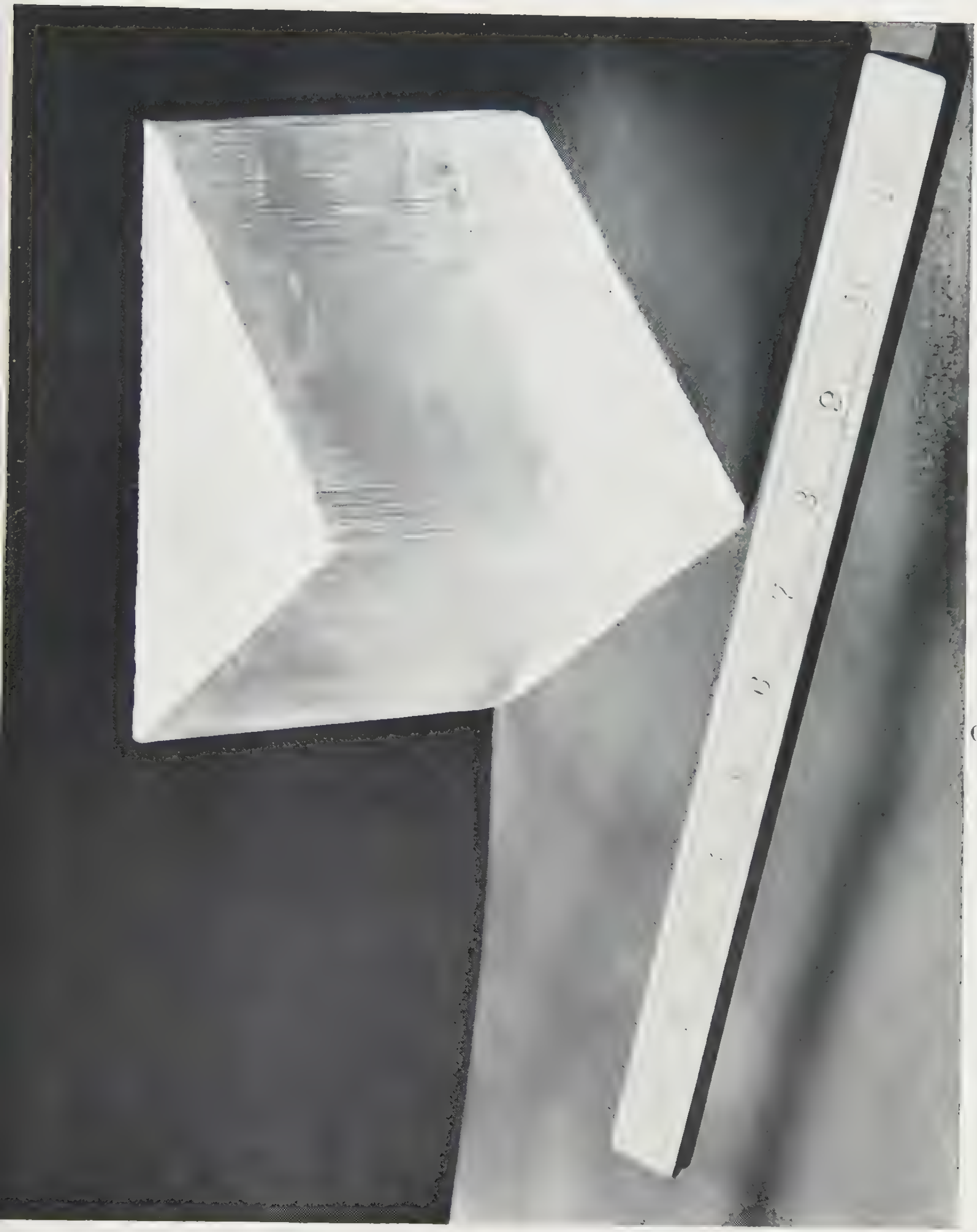

Figure 6. A synthetic rocksalt prism blank with faces 8″ × 5″.

(Courtesy of Harshaw Chemical Co.)

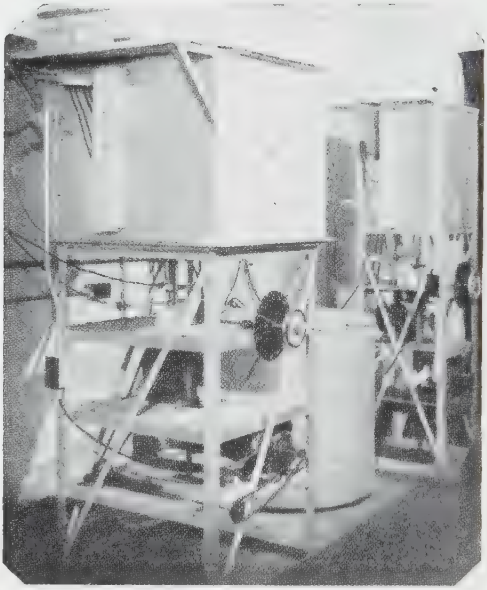

Figure 7 *a*. Furnaces used at the Harshaw Chemical Co., U.S.A., for growing large crystals.

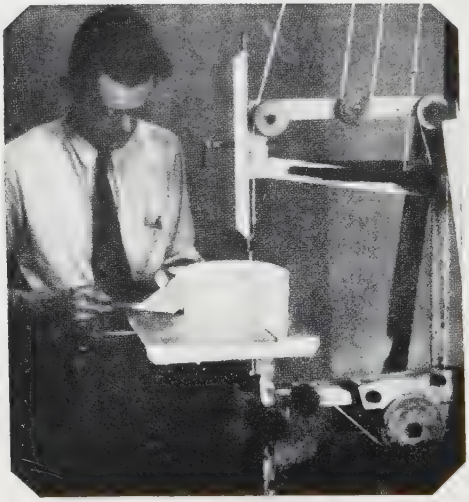

Figure 7 *b*. Cutting large crystals of $NaNO_3$.

(Courtesy of Industrial & Engineering Chemistry.)

are the most difficult to grow, partly on account of the low melting point of the salt. Also crystallization is more rapid in the direction of the optical axis than in other directions; and, further, purification of the salt is difficult. Apparently, however, these various problems have been overcome, and now equally large crystals (i.e. 25 lb. in weight) have been grown. Table 6 gives a comparison of the refractive indices of an artificially grown sodium-nitrate crystal compared with natural calcite. It will be noted from these figures that the refractive indices of $NaNO_3$ are considerably lower than those of calcite, but the birefringence is greater than that for calcite.

It is not without interest to note that the cutting into blanks of such water-soluble crystals is carried out by a wet endless string process (figure 7*b*).

§ 9. PERICLASE

Fused magnesium oxide has been used for some time commercially as a constituent of refractory products and cements, and as an electrical insulator of high thermal conductivity. During the process of manufacture, there separates out on cooling a crystalline product similar in optical properties to the mineral periclase. Crystals exhibit almost perfect cubic formation and occasionally occur as large as 2 in. in face (Strong and Brice, 1935), but the average size is approximately 1-in. cubes. Since the crystals of optical quality occur only as a by-product, the production of them is limited, and not subject to definite control as yet. However, from an optical point of view, it is of interest to note that the effective transmission of the material is from 2300 A. in the ultra-violet up to $5\,\mu$ in the infra-red. It has a high refractive index and fairly high V number (see table 5), which make it valuable as an additional material for the designer of optical systems.

§ 10. MISCELLANEOUS

Apart from the foregoing materials, in which the transmission is sufficient to allow lenses and prisms to be made, there are cases when only small thicknesses of material or liquids are required (e.g. for cell windows or liquid films between lenses), and some information about these latter are given below.

For the ultra-violet region the number of materials available (other than those already mentioned) is small. Thin plates of gypsum and sheet cellophane are useful. A piece of the latter material 0·32 mm. thick will transmit 84 % of the incident light at λ 3132 A., 42 % at λ 2749 A. and 10 % at λ 2313 A. A quartz plate coated with a potassium film of a thickness just sufficient to be opaque to sunlight will transmit from 3000 A. down to 2000 A. (O'Bryan, 1935). More selective ultra-violet filters have been mentioned in the author's 1939 paper. Thin films of liquids which transmit in the ultra-violet are: water, glycerine, ammonia, ethyl alcohol, sugar solutions, chloroform, and ether. In the infra-red region the values for the transmission limits of materials already mentioned, and given in table 5, can be increased (according to the law connecting transmission with thickness) for thin windows; but it is of interest to note that paraffin wax, whilst being opaque up to $12\,\mu$, begins to transmit quite freely from $20\,\mu$ to ∞. Quartz has marked absorption bands from $4\,\mu$ to $45\,\mu$, but thence transmits freely to ∞.

In conclusion, it is hoped that this survey of some recent new materials will stimulate their use in the design of optical and physical instruments. Although we may be as yet only on the fringe of an era of new optical materials, quite a number of advances may be possible with those which are now available.

§ 11. ACKNOWLEDGEMENTS

The author wishes to express his appreciation to Dr. H. C. Kremers (1940), of the Harshaw Chemical Co., Cleveland, Ohio, for his help in supplying much information contained in this paper; also for sending over from America photographs and samples of various crystals for exhibition at this lecture. Many of us in this country will be grateful to him for enabling us to visualize better the progress which has been made in growing crystals of such large dimensions. I would like also to express my thanks to Professor D. C. Stockbarger, of the Massachusetts Institute of Technology, with whom I made contact several years ago, for his continued interest and help in connection with work which I have carried out here, utilizing some of these new materials; and to Mr. W. J. O'Leary, of the Research Laboratories of the Norton Company, Chippawa, Ontario, Canada, for information concerning periclase. Also to Imperial Chemical Industries, Ltd., Messrs. Halex, Ltd., and British Celanese Co., Ltd., for their help and co-operation.

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PLASTICS AND THE OPTICAL INDUSTRY

By W. G. WEARMOUTH,
London

A contribution to a symposium on "Recent Optical Materials" held by the Optical Group 27 *November* 1942

ABSTRACT. Many writers have drawn attention to the possibility of using transparent industrial plastics as substitutes for glass, and some writers have pointed out a few of the main optical defects of these new synthetic materials. Little exact information has so far been published concerning the optical properties of these materials, and the present paper includes a brief account of some recently determined dispersion curves for various plastics.

In addition, a general survey is made of the main transparent plastics, their chief features are described, their main weaknesses from an optical point of view are listed and a table is drawn up which shows how the properties of these materials compare with those of optical glass. Finally, some curves by Frolich are included. These indicate the possibilities of coloured plastics in the field of optical filters.

§ 1. INTRODUCTION

SEVERAL writers have remarked on the similarity in properties of inorganic glasses and the transparent industrial plastics.

Thus, Morgan and his colleagues (1936), in a lecture entitled *Organic Glasses*, gave a very useful description of these transparent plastics, and pointed out how closely they resemble the ordinary glasses. Again, H. Moore (1939), in his paper *Glasses, Organic and Inorganic*, extended the discussions of these materials and drew further attention to their possibilities as substitutes for inorganic glasses. Furthermore, V. E. Yarsley (1938) has written similarly in his article *Plastics and their relation to the Glass Industry*, though he emphasizes that plastics should not be considered as rivals to glass but rather as its willing allies.

Morgan pointed out that organic glasses, to compete with their inorganic counterparts, should possess the following properties: good transparency, freedom from colour, ease of manipulation, stability under extreme weathering conditions of temperature and humidity, low water absorption and high surface or scratch hardness.

He also showed that most of the synthetic resins known at that time possessed some, but not all, of these desired properties. The materials are, for example, very much less stable than glass under severe service conditions, have comparatively low softening temperature ranges, are somewhat water absorbent, and have only moderate surface hardness. On the other hand, they can be easily machined, many of them are exceedingly transparent and a few can be obtained practically colourless. He included a table of physical properties to show how these materials compared with inorganic glasses.

During the six years or so since Morgan's paper appeared, the family of industrial plastics has grown considerably, and the older members of the family have, also through continuous research, been considerably improved.

As a result, we have a fairly large number of transparent plastics which might be considered as possible substitutes for optical glasses. These will now be listed:

Class A
(all based on derivativesof cellulose)
(*a*) Cellulose nitrate plastics
(*b*) Cellulose acetate plastics
(*c*) Cellulose acetobutyrate plastics
(*d*) Ethyl cellulose

Class B
(*a*) Cast phenol-formaldehyde resin
(*b*) Moulded phenol-formaldehyde resin
(*c*) Moulded urea-formaldehyde resin

Class C
(*a*) Polyvinyl acetate
(*b*) Polyvinyl acetals
(*c*) Polyvinyl chloride
(c_1) Polyvinyl chloride acetate co-polymers
(*d*) Polymerized methyl isopropenyl ketone
(*e*) Polystyrenes
(*f*) Polyacrylates

Many of these materials have been discussed in a recent paper (Wearmouth, 1942), and several references to them appear in the literature.

§ 2. CELLULOSE DERIVATIVES

The first group of materials are all derived from the naturally occurring product cellulose. In order to convert these derivatives to tough plastics, however, solvents of high boiling point or semi-solvents (called plasticizers) must be blended with them. Moreover, this mixing is not infrequently done in the presence of some other more volatile solvent, which is subsequently removed in the so-called seasoning process applied to the toughened plastic. It is not always possible to remove the last traces of this solvent (indeed, for many commercial applications of these materials such absolute seasoning is not necessary), and this leads to difficulties from the optical point of view. For example, there is always a tendency for a slight but continuous shrinkage of these plastics to take place in service due to the slow but continuous loss of part of the solvents and plasticizers. This shrinkage is not important for many commercial applications, but it is a disadvantage if the material is to be used for optical work, where the risk of any slight distortion of the finished optical components must be carefully avoided.

These cellulose plastics have only a comparatively low surface and scratch hardness, and they also absorb water, with corresponding changes in dimensions.

From the point of view of transparency, materials of quite good clarity and colour are available in the cellulose range of plastics, but they are not really much superior to standard glass.

Generally speaking, therefore, these cellulose materials have not found very great use for the manufacture of optical pieces.

§ 3. PHENOL-FORMALDEHYDE AND UREA-FORMALDEHYDE RESINS

The clear phenol-formaldehyde and urea-formaldehyde resins in class B are produced by the interactions of phenol and formaldehyde and of urea and formaldehyde respectively. Both reactions must be carried out under carefully controlled conditions and in the presence of suitable catalysts. Transparent resins may be thus produced which can be either cast or moulded (in the case of the phenol-formaldehyde type) or simply moulded (in the case of the urea type).

These resins have much to commend them: they possess a comparatively high scratch resistance, though not as high as that of glass; they are practically infusible, i.e. they have no measurable softening temperature, though they char at temperatures above 180° C.; and they show a low coefficient of expansion in comparison with the cellulose derivatives. They can be polished fairly easily or they can be made to take up accurately the shape of a given mould. Yet they are not proving very popular in the optical field.

In the case of the urea-formaldehyde resins, this unpopularity is probably due to their comparatively high water-absorption coefficient and probably, to a certain extent, to the comparatively high cost of the resins.

With the phenol-formaldehyde resin, where the water absorption is lower, the chief objection probably arises from the fact that the production of mouldings entirely free from internal strain is somewhat difficult.

Owing to the rather brittle nature of these plastics, it is not always easy to grind perfect optical surfaces on them. Moreover, it is somewhat difficult to get these plastics entirely free from colour; and even with the best colours there is a tendency for the colour to deteriorate on exposure of the plastic to severe climatic conditions.

§ 4. POLYMERIZATION PRODUCTS

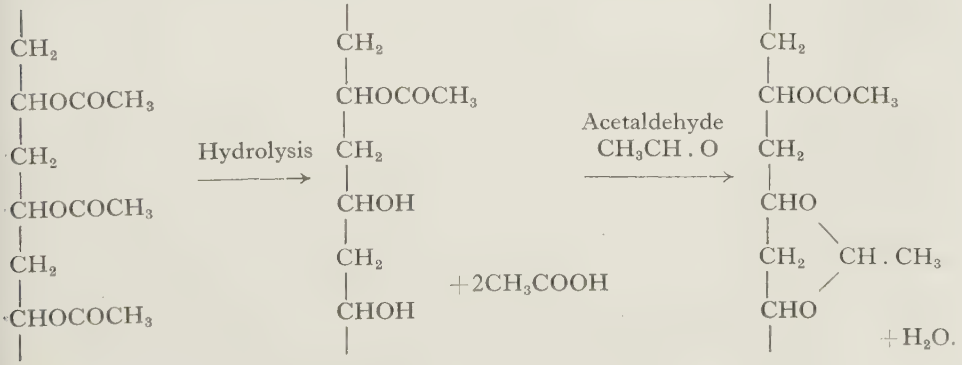
We come then to the class C types of plastic, the so-called polymerization products.

Polyvinyl acetate

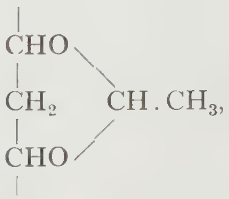
Polyvinyl acetate is probably the simplest, and is the polymer of vinyl acetate, a colourless liquid of $\mu_D = 1{\cdot}396$. It has excellent clarity and is colourless. Unfortunately, it suffers seriously from a tendency to cold flow, and it is not, therefore, being put forward as a material for optical work.

Acetals

The substituted polyvinyl acetates, called polyvinyl acetals, are not so flexible as the parent substance, and may, therefore, be considered as possible materials for optical work. If polyvinyl acetate is partially hydrolyzed to remove some of the acetyl groups, and the exposed hydroxyl groups are then caused to react with aldehydes, the polyvinyl acetals are formed:



The above scheme shows the formation of an acetal with acetaldehyde from vinyl acetate The resulting resin is available under the trade name of " Alvar ". Note the ring formation,



in the resin structure. This may account for the strength of these resins (Hetherington, Couzens and Turner, 1940).

The following trade names are known and correspond to the acetals formed from the aldehydes stated:

Formvar from formaldehyde	$HCHO$
Alvar from acetaldehyde	CH_3CHO
Butvar from butylaldehyde	C_3H_7CHO or $CH_3 . CH_2 . CH_2 . CHO$

These polyvinyl acetals are much less liable to cold flow than the straight polyvinyl acetate. They are not, however, entirely free from colour, though it is stated that they can be more specially treated to give resins which are practically colourless. So far, however, these resins do not appear to have been employed very extensively, if at all, for optical work.

They have been chiefly used so far as the interlayer for safety glass. The resins are tough, have a high surface hardness and have comparatively low water-absorption coefficients.

Polyvinyl chloride

Polyvinyl chloride based on vinyl chloride, $CH_2 : CHCl$ (a colourless gas), is a similar polymerization product. It is colourless and has a comparatively high softening point. Because of this last-mentioned property it is often used in conjunction with plasticizers, which naturally impart flexibility and elasticity.

These induced properties tend to make the material of greater use as a coating composition or as flexible sleeving. It has not, therefore, been used to any large extent for optical work.

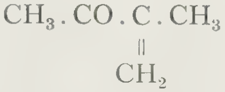
Vinyl acetate-vinyl chloride co-polymers

The co-polymer of vinyl chloride and vinyl acetate is not unlike the polymers of the basic materials. Indeed, it is stated that co-polymers can be made which possess the best properties of the two basic polymers. Many co-polymers are known; the materials are transparent, and some can be obtained in fairly hard grades. Despite these somewhat attractive properties, the polymers have not been proposed as materials for optical work. This may be due to the great demands for this material for use in electrical insulation work.

Polymerized methyl isopropenyl ketone

An interesting transparent material is the resin discussed by K. W. Pepper and his collaborators (Morgan *et al.*, 1938) and based on polymerized methyl isopropenyl ketone.

Methyl isopropenyl ketone,



$$CH_3.CO.\underset{\underset{CH_2}{\|}}{C}.CH_3$$

(or methylene methyl ethyl ketone), a colourless liquid, $\mu_D = 1{\cdot}422$, is formed when a mixture of methyl ethyl ketone and formaldehyde is allowed to react under suitable conditions. The monomeric liquid can be made to polymerize under controlled conditions to give a hard, clear, transparent resin with excellent mechanical properties, comparatively low water-absorption coefficient, and a moderate softening temperature range. The refractive index is high at 1·52 and the material has a moderate relative dispersion of 0·0098.

Polystyrenes and acrylates

Rigid polymeric materials are found in the two groups: polystyrenes and polyacrylates.

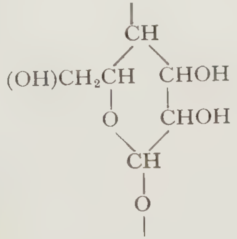
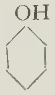
Polystyrene (trade names: Styron, Distrene or Monsanto Polystyrene) is, of course, the polymer of styrene, $C_6H_5.CH{=}CH_2$ (vinyl benzene), which is itself a mobile and colourless liquid of refractive index $\mu_D = 1{\cdot}543$. The polymer is colourless and quite transparent; it is a hard and rigid material, though its surface hardness is somewhat low.

Its softening-temperature range is only moderate, but the material has the advantage of being impervious to water. Indeed, its water-absorption coefficient is stated to be nil. These properties, combined with the chemical inertness of the material (it is, for example, only affected by hydrocarbon solvents and chlorinated hydrocarbons), make it an obvious choice for optical work. It can be moulded to give strain-free mouldings—although care is needed here—and it can also be cut and polished. The coefficient of expansion is fairly high, and hence care must be taken when using this material at elevated temperatures, especially if the optical pieces are used in conjunction with metal mountings.

The polyacrylates are the final class of transparent resins with which we will deal. Polymethyl methacrylate, the one most commonly employed (trade name "Perspex") is stated to be formed from the monomeric material methyl methacrylate $CH_2 = C.CH_3\,COOCH_3$ (acrylic acid is $CH_2 = CHCOOH$ and methacrylic acid $CH_2 = C(CH_3)COOH$).

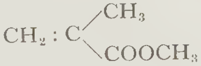
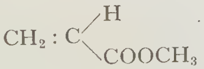
This monomer is a clear transparent liquid of refractive index 1·418, and it polymerizes quite readily to give resins varying from soft, sticky semi-liquid, to hard, tough, thermoplastic solids, according to the conditions and degree of polymerization. The resins are very transparent and colourless, possess a fairly high softening-temperature range and have good weathering properties. They can be moulded and have been proposed as useful substitutes for optical glasses. An English company claims to have made excellent lens systems from them. The surface hardness of the resins is not high, but again claims are made

Table 1. Properties of typical plastics

Material	Basic material(s)	Transparency and ageing	Refractive index (typical values)	Mechanical strength	Hardness	Water absorption (24 hours immersion) (%)	Softening point (° c.)	Coefficient of expansion × 10^{-5} per °c.	Remarks
Cellulose nitrate	 + HNO_3	Very good and quite stable	1·50	Tough. Rather flexible	8–11	1–3	71–91	12–16	Flexible material; can be machined or moulded. Shows a gradual shrinkage due to loss of volatile constituents. This prevents its use as an optical glass.
Cellulose acetate	Ditto + CH_3COOH	Very good and quite stable	1·48	Ditto	9–12	2–3	60–71	14–16	Flexible material, etc.; absorbs atmospheric moisture somewhat readily.
Ethyl cellulose	Ditto + C_2H_5Cl	Moderately good and stable	1·47	Ditto	5–10	0·5–1·5	60–82	10–15	Flexible material, etc.; more resistant to moisture than the acetate.
Cellulose acetobutyrate	Ditto + C_3H_7COOH CH_3COOH	Quite good and stable	1·49	Ditto	8–15	0·6	60–82	10–15	Flexible material, etc.; water resistance better than acetate.
Phenol-formaldehyde. Cast	 + HCHO	Good. Moderately stable	1·60	Rigid. Rather brittle	30–40	0·1–0·5	None	2–3	Transparency good, though colour is not too stable. Rather brittle material. Has to be polished like glass.
Moulded phenol formaldehyde	Ditto	Ditto	1·59	Ditto	Ditto	Ditto	Ditto	2–3	As cast material except for moulding properties. Difficult to get stress-free mouldings with high finish.

Moulded urea and thio urea formaldehyde	$CO(NH_2)_2 + HCHO$ $CS(NH_2)_2 + HCHO$	Very good, but absorbs moisture	1·60	Ditto	48–50	1·0–2·0	Ditto	1·8	Similar to moulded phenolics. Water absorption a disadvantage.
Polyvinyl acetate	$CH_2 : CHOCOCH_3$	Very good. Suffers from cold flow	1·48	Flexible. Liable to cold flow	Low	Up to 2·0	Fairly low	6–8	Transparency good. Tendency to cold flow a serious disadvantage.
Polyvinyl acetals (Formvar, Alvar, Butvar)	$CH_2 : CHOCOCH_3 + HCHO$ Ditto $+ CH_3CHO$ Ditto $+ C_3H_7CHO$	Moderately good	1·52 1·52 1·52	All rigid materials	30–45 Ditto Ditto	Ditto Ditto Ditto	100 80 90	*c.* 10 *c.* 10 *c.* 10	These three materials fair. Shrinkage due to loss of plasticizer is a disadvantage.
Polyvinyl chloride	$CH_2 : CHCl$	Reported good	1·52	Rigid or flexible	2–50, depending on grade	0·5–1·3	66–72	6–8	Usually too flexible. Also shrinkage due to loss of plasticizer is a disadvantage.
P.V.C. & P.V.A. co-polymers	$CH_2 : CHCl$ $CH_2 : CHOCOCH_3$	Ditto	1·53	Ditto	Ditto	Ditto	66–72	6–8	Similar to polyvinyl chloride.
Polymerized methyl isopropenyl ketone	$CH_3COC\,.\,CH_3$ $\|$ CH_2	Very good. Moderately stable	1·52	Rigid. Non-brittle	20	0·3	60–80	6–8	Hard rigid material. Can be moulded. Chief disadvantage is the slight colour change on exposure to light.
Polymerized styrene	$C_6H_5CH : CH_2$	Excellent. Very stable	1·59	Rigid. Rather brittle	20–30	0·05	*c.* 60	6–7·5	Excellent stability. Hard and rigid material, but rather brittle. Can be moulded into stress-free mouldings. Transparency as good as glass. Disadvantage: low surface hardness. Negligible water absorption. Material prepared without plasticizer shows no distortion on ageing.

Table 1. Properties of typical plastics (*continued*)

Material	Basic material(s)	Trans-parency and ageing	Refrac-tive index (typical values)	Mechanical strength	Hard-ness	Water absorption (24 hours immersion) (%)	Soften-ing point (° c.)	Coefficient of expan-sion × 10^{-5} per °c.	Remarks
Polymerized methyl meth-acrylate	 $\mathrm{CH_2 : C}\begin{matrix}\diagup \mathrm{CH_3} \\ \diagdown \mathrm{COOCH_3}\end{matrix}$	Excellent. Very stable	1·50	Rigid. Tough	18–20	0·4	60–70	7–9	Quite tough and rigid material, excellent stability. Can be moulded. Transparency excellent. Can be cast or moulded in stress-free form. Practically no water absorption. Stability as for polystyrene. Disadvantage : low scratch resistance.
Polymerized methyl acrylate	 $\mathrm{CH_2 : C}\begin{matrix}\diagup \mathrm{H} \\ \diagdown \mathrm{COOCH_3}\end{matrix}$	Ditto	1·51	Ditto	Ditto	0·4	Ditto	Ditto	Similar to polymerized methyl methacrylate.
Optical glass	—	Excellent. Stable	—	Rigid. Somewhat brittle	Very high	Nil	Very high	Usually less than 1·0	Excellent properties, but rather brittle.

that, by special treatment, lenses with almost glass-hard surfaces can be obtained. Briefly, in this special process a hard coating is applied to the lens after manufacture. The lens to be coated is placed in a chamber which can be evacuated and the vapours from a heated bath of silica or glass are allowed to impinge on the surface of the lens. The vapours condense to give a hard and permanent glass-like coating on the lens.

There are other acrylic resins of very similar structure and properties; for example: polymethyl acrylate $(CH_2:CHCOOCH_3)_n$, polymethacrylic nitrile $(CH_2:CCH_3CN)_n$, and the polymers of a number of esters of methacrylic acid $CH_2:CCH_3COOH$; but for the present the polymethyl methacrylate resins appear to be those which have found most favour in the plastics industry for optical work.

It will be convenient at this stage to draw up a table showing these available transparent plastics and their chief physical properties. This is given in table 1.

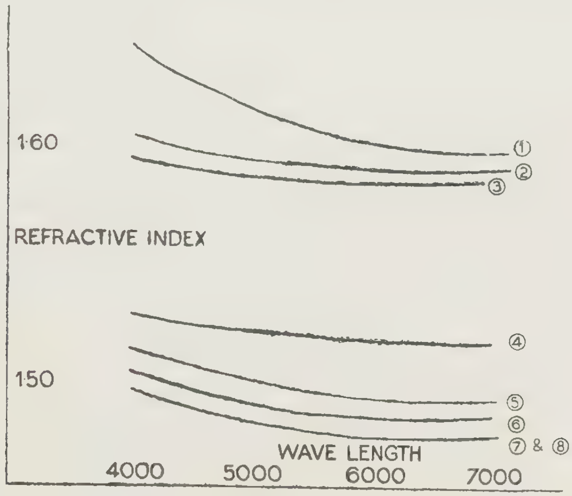


Figure 1.

(1) Cast phenol formaldehyde ("Runcolite" grade 1).
(2) Polystyrene (U.S.A.).
(3) Polystyrene ("Distrene").
(4) Polymethyl isopropenyl ketone (after K. W. Pepper).
(5) Celluloid (cellulose nitrate plastic).
(6) Polyacrylate resin I.
(7) Cellulose acetate ("Novellon").
(8) Polymethyl methacrylate (II).

The figures in this table show that the most suitable plastics for optical work re the polymethacrylates and polystyrenes. These materials are water-white nd have excellent clarity. Moreover, they suffer little deterioration in colour n prolonged exposure to light or to moist atmospheric conditons. They are quite rigid, can be moulded, and when unplasticized do not show any appreciable limensional changes on exposure to atmospheric conditions for long periods.

The surface of these two resins is, unfortunately, easily scratched; but, as lready stated, a process has been worked out for covering pieces of acrylic resin vith a hard protective coating, and it is believed that styrene resins can be treated n a similar manner.

The only remaining point for discussion is the possibility of producing chromatic lenses from plastics. It may be worth while examining some typical dispersion curves which have been obtained in the laboratories of the South-West Essex College for a number of industrial plastics. These are shown in figure 1, in which there are two curves for two grades of polystyrene and two for two polymethacrylates.

It should therefore be easily possible to work out combinations of polystyrene lenses and acrylate lenses to give achromatic systems using the standard procedure. It is also clear that achromatic lenses can be prepared using the two types of acrylates on the one hand or the two types of polystyrenes on the other.

This idea has been extended by K. Frolich (1940) in a recent paper, from which dispersion curves for six different resins of the acrylic type are reproduced in figure 2.

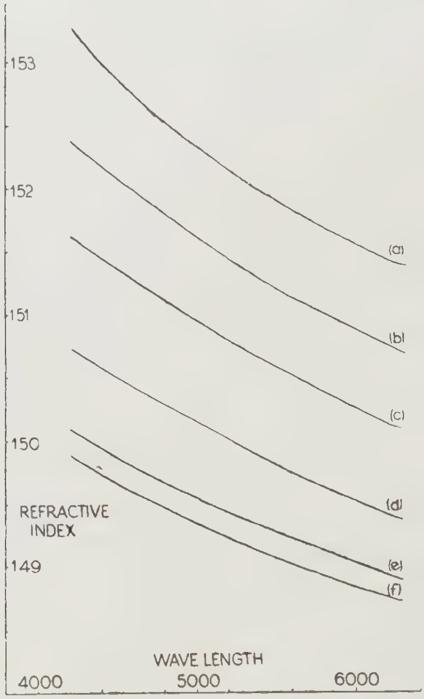
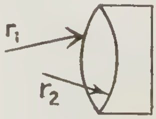


Figure 2. Dispersion curves for six types of Plexiglas (after Frolich).

It is interesting to note that the refractive index of liquid methyl methacrylate monomer is 1·417 and that of the polymer 1·498 (as determined in the South-West Essex College Laboratories), i.e. there is an increase in refractive index with polymerization.

It will be seen that achromatic systems can be built up using any two of these materials, and Frolich goes on to calculate the lenses required of material type (*a*) and type (*f*) to give an achromatic system with a focal length of 10 cm.

The radii are as follows:—



$r_1 = 14{\cdot}98$ cm.
$r_2 = 13{\cdot}62$ cm.

He concludes that it should be possible to construct achromatic systems using the same basic plastic for the two components. The same is probably true of the polystyrenes and some of the other polymerized substituted-acrylates.

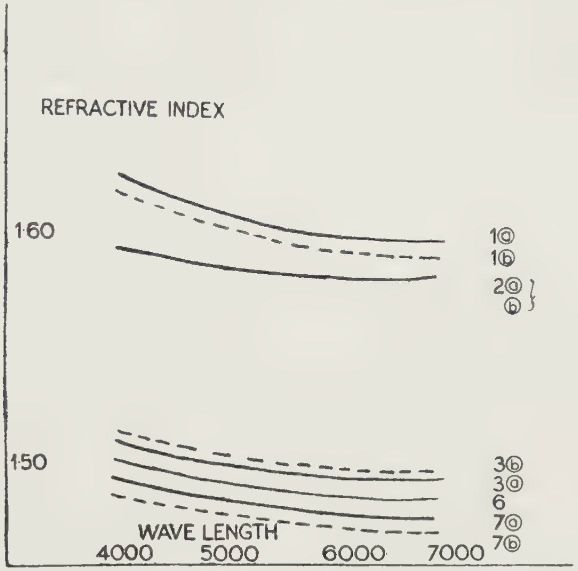


Figure 3.
(*a*) Before immersion in water ; (*b*) after immersion in water.

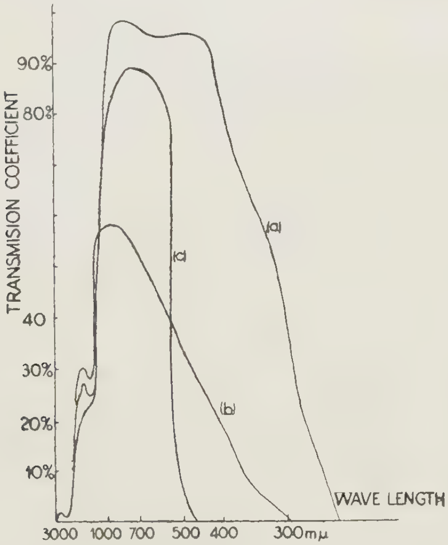


Figure 4.
(*a*) Plexiglas M222 (5 mm. thick) ; (*b*) Plexiglas M236 (grey) (3 mm. thick) ; (*c*) Plexiglas M71 (yellow) (3 mm. thick).

The curves shown in figure 3 are also interesting, since they show the effect on the refractive index when the plastic is immersed in water for 24 hours. Methyl methacrylate and styrene (curves 6 and 2) show no change, though the curves for cellulose acetate (7), cast phenolic (1), and cellulose nitrate (3) show marked changes after similar treatment.

§ 5. COLOURED FILTERS

Frolich also describes the transmission curves for certain acrylic types of resin, and three of these curves are shown in figure 4. It will be seen from these curves that the materials transmit up to 3 μ and cut off at about 270 mμ; the second curve for a coloured material (grey) shows a cut off at 380 mμ in the ultra-violet with the infra-red transmission still up to 2·5 μ.

Frolich claims that these materials can be used for accurate optical filters, and it is indeed a fact that the polymerized acrylates appear to be very stable chemically and to have few, if any, effects on many dyes incorporated in them.

This property explains why it is relatively easy to make up tinted or delicately coloured varieties of the polyacrylates or polystyrenes to match existing standard colour plates.

There is probably a very big future for this type of filter based on the so-called organic glasses.

ACKNOWLEDGEMENTS

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DISCUSSION

Prof. L. C. Martin. Can Dr. Wearmouth give us any information about the homogeneity of plastics as regards their expansion coefficients with temperature changes and their capacity to retain a constant shape under varying conditions of temperature?

Absorption of moisture might be almost negligible under ordinary conditions of measurement, but might possibly produce a relatively large effect in the condition of the material near the surface. One does not doubt the usefulness of the material for optical purposes such as spectacles, where the figure need only be moderately good, but in highly corrected systems any local variations of surface stresses and distortions of the material may be disastrous. Has any work on this question been published? Are plastics liable to such effects?

Capt. T. Y. Baker, R.N. There is one feature connected with the commercial production of moulded plastic lenses which merits very serious attention by optical designers. It seems likely that in the very near future it will be possible to mould lenses with non-spherical curves at a price which is much lower than would be necessary if the lenses were made of glass. The latter would have to be ground and polished one at a time by a highly skilled worker. Now lenses of non-spherical form enable the optical designer to modify aberrations without interfering with the paraxial image formation. To take a concrete example, consider the case of an image formed by a spherical surface of radius r. This surface is formed by the revolution round the axis of the circle $y^2=2rx-x^2$. If we replace the circle by the conic $y^2=2rx-px^2$, we still get the same paraxial image formation, but the parameter p enables the designer to vary the aberrations over a wide range. For instance, if r be taken as unity and the refractive index is 1·5, the image of a distant object is formed at a distance 3 beyond the surface. With a finite aperture the spherical aberration is under-corrected (for the sphere). Taking a ray at a height 0·5 and varying values of p, the longitudinal spherical is given below :

$p=$	-2	-1	0	1	2	4
long. sph.	0·853	0·544	0·205	−0·175	−0·570	−1·697

Thus it will be seen that somewhere between the paraboloid ($p=0$) and the circle ($p=1$), there is an ellipsoid that will give freedom from spherical. Actually the value is $p=1-1/n^2$ and all rays are exactly corrected.

This simple example will show how great an advantage the designer can get by his ability to utilize non-spherical surfaces. The point has, of course, been known for many years, but no extensive design work has been done in this direction, as the inability to produce them on a commercial scale has made the arduous labour involved not worth while. The time seems to have come when designers should make a start, especially among photographic lenses.

Another direction in which moulded plastics might be of use is in the complicated prism systems that occur in many modern instruments. But to do this satisfactorily requires that the reflecting surfaces should come from the mould absolutely flat. For a curved surface small alterations of curvature can be dealt with by re-focusing. The faintest trace of curvature in a reflecting plane would be fatal. It would be interesting to have from Dr. Wearmouth some statement of the exact quality of optical flats that can be moulded.

AUTHOR'S reply. I understand that it is possible, with care, to reproduce by moulding any given surface, and if this information is correct it would seem that the two projects mentioned by Captain Baker are quite practicable in transparent plastics.

In reply to Professor Martin, provided the temperature range envisaged does not exceed the softening temperature range of the plastic, no practical distortion of the plastic will occur, although some change of dimensions with large changes of temperature are bound to occur, as emphasized in the paper.

There is no evidence, as far as the writer is aware, of serious changes in or near to the surfaces of the polyacrylic resin and polystyrene optical pieces when they are immersed in water. The other transparent plastics are, however, not so resistant to water. The special hardening surface treatment given to the two specially-mentioned resins may have an effect on their water-absorption properties, but no published information is available on this point.

PLASTIC SPECTACLE LENSES

By H. H. EMSLEY,
Northampton Polytechnic, London

A contribution to a symposium on "Recent Optical Materials" held by the Optical Group 27 *November* 1942

ABSTRACT. Perspex, a polymer of methyl methacrylate, possesses all the attributes, except hardness, desirable in a material for spectacle lenses. It is especially resistant to impact and splintering and is extraordinarily light in weight. Experiments are described illustrating (*a*) that surface scratches, arising because of the relative softness of Perspex, are not so troublesome to vision as might be supposed ; (*b*) the resilience and resistance to splintering of Perspex. The manufacture of plastic lenses, in which the lapping operations necessary with glass are replaced by a simple moulding operation, is briefly described. Plastic lenses for binoculars, telescopes and photography, as well as plastic spectacle lenses, are in production.

§ 1. PERSPEX

Attempts have often been made to find substitutes for glass for this or that optical purpose. Although glass has remained supreme, it is not free from certain disadvantages. The materials required for its manufacture are not widely distributed; its manufacture is by no means a simple process; it is very brittle, and the fragments into which it breaks are often dangerous. Any material evolved to possess the essential good properties of glass, and in which some or all of its defects were eliminated, would be a most desirable material.

The wonderful array of plastic materials being produced nowadays by the chemists include certain types that bid fair to possess practically all the desirable features. The general properties of the transparent group of truly synthetic resins, especially the styrol and methacrylic acid types, have been adequately described in the papers by Dr. Wearmouth and Mr. Johnson. The latter type appears to be admirably adapted for the manufacture of specacle lenses; and it is with certain aspects of this particular application that I propose to deal.

I am informed that plastic spectacle lenses will be generally available in the early part of 1943. They are made from methyl methacrylate, a derivative of methacrylic acid, the trade name being Perspex. An inspection of the samples of finished spherical and toric lenses that have been kindly loaned by Messrs. Clement Clarke, Ltd., will reveal their exceptional lightness in weight, their remarkable transparency and their toughness; it is indeed not easy to break them. The material is very inert chemically and has no toxic properties: contact with sensitive skins produces no form of dermatitis. Perspex is very stable: it retains its mechanical properties, its transparency and surface finish under a wide range of climatic and temperature changes. Indeed, for spectacle lenses, and for other optical elements, Perspex possesses practically all the desirable qualities. In only one important feature is the material, at present, lacking;

it is rather soft, and so scratches much more readily than glass. Naturally the problem is receiving attention, and improvement in this respect is confidently expected. I refer to this problem later.

The properties of Perspex that are of optical interest are given in the accompanying table, with some comparative figures for glass.

Some properties of Perspex

	Perspex	Some comparative figures for glass	
Specific gravity	1·19	2·3 (crown) to 5·9 (flint)	
Water absorption, in 7 days at 20° c.	0·3 to 0·4%		
Water condensation on surface : time required to disappear	2·5 minutes	3·5 minutes (plate)	
Refractive index N_D	1·495 to 1·490	1·523	Spectacle crown
V value $\dfrac{N_D-1}{N_F-N_C}$	53·3 to 58	52–56	
Light transmission—visible	92%	91·5%	
Ultra-violet 340 mμ	82% (2·3 mm. thick)	80%	
,, 310 ,,	62 ,,	25 ,,	
,, 290 ,,	36 ,,	0	
Infra-red 1000 ,,	90% (5 mm. thick)	90%	
,, 1400 ,,	55 ,,	90 ,,	
,, 2000 ,,	50 ,,	90 ,,	
,, 2200 ,,	12 ,,	87 ,,	
,, 2500 ,,	8 ,,	85 ,,	
Tensile strength, at 20° c.	3·2 to 3·9 tons/in.^{2}		
Ultimate fibre stress in bend, 20° c.	6 to 7 tons/in.^{2}		
Impact strength—Izod, cut notch 20° c.	3·2 to 4·2 cm.-kg.		
Impact strength—Izod, cut notch 40° c.	4·3 cm.-kg.		
Brinell hardness	18 to 21		
Heat resistance, Marten's	55–60° c.		
Thermal conductivity	5×10^{-4} c.g.s. units	$(15\text{ to }25)\times10^{-4}$ c.g.s. units Copper about 1·0 unit	

§ 2. PROPERTIES REQUIRED IN A MATERIAL FOR SPECTACLE LENSES

In relation to the essential properties of transparency to visible light and stability, lenses made of glass and of Perspex are more or less equally efficient.

There are other properties, not so fundamentally essential, but to varying extents desirable, such as:

(*a*) lightness in weight ;
(*b*) opaqueness to short-wave ultra-violet and near infra-red ;
(*c*) clearing of surface condensation ;
(*d*) hardness;
(*e*) resistance to impact ;
(*f*) resistance to splintering ;
(*g*) accuracy, ease and convenience of working and manipulation.

Under (*a*), (*b*) and (*c*) Perspex is, for ordinary everyday wear, superior to glass. Lightness in weight is specially advantageous in connection with the application to contact lenses, and plastic contact lenses are, in fact, already being made.

§ 3. HARDNESS: THE VISUAL EFFECT OF SCRATCHES

There is no question that Perspex is softer than glass. Although the moulding of plastic lenses is followed by a "hardening" process applied to their surfaces, the latter are definitely scratched more easily than the surfaces of glass lenses. In practical use this greater susceptibility to scratching does not, however, constitute a drawback of as much significance as might be supposed. In the first place, the scratches on Perspex are fine and smooth and do not exhibit the irregular fractured appearance found in a scratch in a brittle material such as glass. Secondly, it might be argued that the standard of surface "purity" that is traditionally imposed when dealing with glass lenses is a somewhat artificial one. It is easy to maintain the standard with glass lenses and easy to direct the attention of the customer to the impeccability of their surfaces, since he is not normally in a position to examine defects of much more practical importance, such as accuracy in power, axis direction, etc., defects which are always present in some degree. Tests that I have carried out reveal that the effect of scratches on the wearer's vision and comfort is much less than is ordinarily supposed.

A scratched surface interferes with vision by scattering the light incident on it and thereby reducing the contrast between the object under observation and its background, the effect being enhanced when the background is bright relative to the object or when it contains local bright sources. To test this effect a flat plano plastic disk of 3 mm. thickness was deliberately scratched with a suitable fine abrasive which produced scratches of somewhat similar fineness to those that appear on a plastic lens after a period of wear; if anything the scratches on the test disk were coarser. In order that the effect on vision should be appreciable and capable of reasonably accurate measurement with simple apparatus, the number of scratches was, as the illustration (figure 1) indicates, immeasurably greater than the number to be expected, even after months of careless wear. Figure 1 is a photograph of the actual specimen.

The effect of this scratched specimen on vision was tested by measuring the brightness contrast sensitivity (Weber fraction) of the eye, firstly when unaided and secondly when provided with the specimen. The arrangement of the apparatus is indicated in figure 2. S is a matt white screen; A and B two 100-c.p. Pointolite lamps; D an obstacle in the form of a wire of about 1/8-inch diameter bent into some geometrical shape—triangular, circular, rectangular, etc. The observing eye at E was shielded from the lamps and prevented from seeing D by means of the screen C. The eye observed steadily the region F, where the shadow of D due to lamp B was to appear. Lamp A was so positioned that the shadow it cast was beyond the field of vision of the eye; the distance AF was approximately 40 cm. Thus the shadow at F was illuminated only by lamp A, whereas the surrounding parts of the screen S were illuminated by both A and B. Three different shapes of obstacle were placed at random at D, the observer being unaware as to which shape was in position at any given time. The lamp B was steadily approached from a considerable distance, exceeding 3 metres, until the observer, gazing at F, recognized the existence of the shadow and successfully named its shape. Distance BF was then recorded. As the result of several readings with three observers, the average value of BF was 295 cm. for the unaided eye and 283 cm. for the eye provided with the scratched plastic specimen. The average illumination on screen S, measured with a lumeter, was 470 metre-candles.

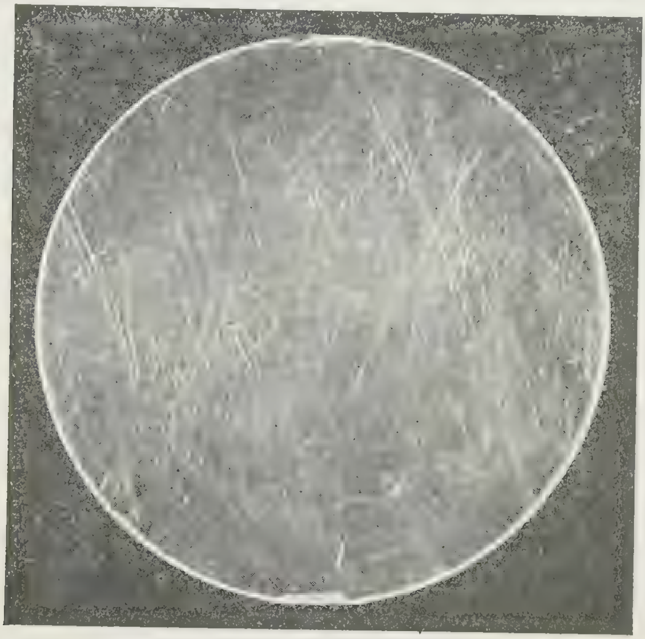


Figure 1. Photograph of scratched plastic disk.

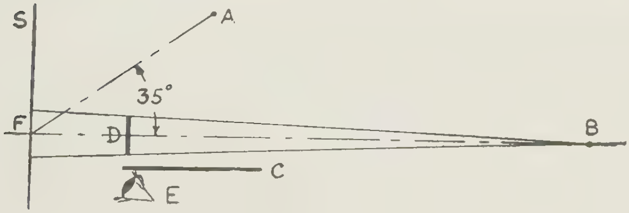


Figure 2. Apparatus.

If E_A = illumination at F due to lamp A alone $= I_A \cos\theta / d_1^2$

and E_B = illumination around F due to lamp B alone $= I_B / d_2^2$,

where I_A and I_B are the candle-powers of the lamps; then

$$\text{Contrast sensitivity} = \tfrac{1}{2}(E_A + E_A + E_B)/E_B$$

$$\text{or C.S.} = E_A/E_B \text{ approx.}$$

$$= (d_2^2/d_1^2)\cos\theta, \text{ assuming } I_A = I_B.$$

For unaided eye, $\text{C.S.} = (2{\cdot}95^2/{\cdot}4^2) \times {\cdot}82 = 44{\cdot}6$.

For eye armed with specimen, $\text{C.S.} = (2{\cdot}83^2/{\cdot}4^2) \times {\cdot}82 = 41{\cdot}05$.

Thus the interposition of the scratched plastic specimen reduced the ability of the eye to appreciate contrast by rather less than 9 %. It is unlikely that a plastic spectacle lens would become scratched by more than a fraction of that of the specimen used in the experiments, even after prolonged wear, and so we might conclude that contrast sensitivity would be reduced by some 2 or 3 % by an ordinarily scratched plastic spectacle lens.

§ 4. RESISTANCE TO IMPACT AND SPLINTERING

This was tested in a manner similar to that laid down by the Ministry of Aircraft Production in their Material Specification D.T.D. 222, August 1940, relating to *Safety glass for plane goggle and spectacle glass.* Three flat planos were used, made respectively of Perspex, ordinary spectacle crown and a well known make of safety glass; each plano was 50 mm. in diameter and 3 mm. thick. Each specimen was held horizontally between two heavy blocks of lead, leaving a clear circular aperture of diameter $1\frac{1}{2}$ in. A steel ball 17·3 mm. in diameter and weighing 21·5 gm. was dropped vertically on to the specimen from gradually increasing heights; these started at 20 cm. and increased in 10 cm. steps until the specimen was fractured. The holding and release of the ball was carried out with the aid of an iris diaphragm. The safety-glass specimen broke when the 60-cm. drop was reached; the glass specimen broke into a large number of large and small pieces at the 90-cm. drop; in the case of the Perspex specimen nothing happened until the 120-cm. drop was reached, at which stage a crack marked (1) in figure 3 developed. Up to this stage the ball bounced after each drop to a height equal approximately to two-thirds of the height from which the ball had been dropped. The height of this rebound was reduced considerably after the crack had developed. A second crack (2), roughly perpendicular to the first, appeared at the 150-cm. drop; and the crack marked (3) at the 200-cm. drop. The dropping from this height of 200 cm. was continued five times, when the specimen broke, the piece marked A falling out in two portions.

From the evidence provided by these three specimens there is no question of the marked superiority of Perspex with respect to the resistance to fracture. I would add that only one specimen of each material was tested, and I do not know whether the same relative figures would always be obtained. The results confirm others which, I am given to understand, have been obtained officially in various ways, but have not been published.

It is to be noted also that the plastic specimen did not splinter; it merely broke into three large pieces, the edges of which were not sharp. The glass specimen splintered into a very large number of sharp-edged fragments of all sizes. The safety-glass specimen cracked, but, of course, there were no free splinters.

These two properties of resistance to impact and to splintering are clearly of great importance and constitute an unmistakable advantage of plastic material as a medium for the manufacture of spectacle lenses and goggles. Plastic lenses are much "safer" than glass lenses.

§ 5. MANUFACTURE OF PLASTIC LENSES

It will be recalled that Perspex is a true synthetic resin, being a polymer of methyl methacrylate, and that it is of the thermoplastic class. The polymerization process is so controlled as to occur during casting in suitable moulds, producing sheets, rods, tubes, etc. Perspex intended for the manufacture of lenses is supplied in sheets of standard sizes and thicknesses, these sheets being the raw material to the lens manufacturer.

Although Perspex can be cut and machined in various ways with the ordinary type of engineering tools and cutters, its pre-eminent characteristic is its pliability and freedom from brittleness, so that it can be easily moulded into almost any desired shape under suitable conditions of pressure and temperature. It is this property that is utilized in the manufacture of lenses.

In the first place, steel dies of the required spherical or toroidal curvatures are prepared. After careful machining, these dies are subjected to successive grinding and polishing operations until their surfaces are comparable in their degree of polish to the surfaces found on lenses. To enable the requisite high standard of accuracy and polish of these dies to be achieved, special steels are employed, and considerable experimentation has been necessary to decide upon the abrasives and grinding operations to be employed. However, when the two dies required for the moulding of a lens of given surface curvatures have once been made, they remain available, with only occasional light polishing, for a considerable period of time and continue to impart their curvature accurately and constantly to a very large number of lenses. The surfaces of the dies are protected against atmospheric and other corrosive influences by means of a thin film of oil. Their peripheries are of a definite shape and size, so that when placed in the moulding press they take up a definitely oriented position relative to the centre line of the press.

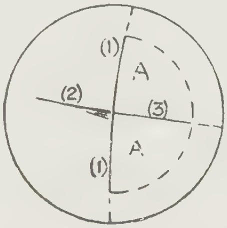


Figure 3.

The cavities in the press that receive the dies are steam-jacketed. The pressure of the dies on the plastic material and the temperature of the steam in the jackets are under the control of the operator, the required valves and gauges being situated on a panel within comfortable reach and vision.

The operations needed to produce a finished plastic spectacle lens are surprisingly few. Circular disks are first cut from a sheet of Perspex of appropriate thickness by a trepanning tool operated in a vertical milling machine. The surfaces of the disk are then turned on a lathe to curvatures approximating to those finally required. Dies of the appropriate curvatures having been drawn from the store, the turned lenses are moved to the moulding room, which contains batteries of presses, severally adapted for various types and sizes of lenses. The moulding operation follows.

It must not be thought that the lenses are moulded with the kind of rapidity one often sees in the case of power presses for stamping out metal parts in an engineering workshop, where the press, operated by a boy or girl, turns out the parts at the rate of ten or thiry per minute. For each plastic lens the dies must first be raised to the correct operating temperature by passing steam around the jackets; this temperature varies according to the form and thickness of the lens. When the required temperature is reached, the upper die is lowered and the lens pressed between the two dies with the proper pressure, the temperature and pressure being maintained for a period of two, three or four minutes, depending on the conditions. The pressure is maintained for a further period whilst the temperature is reduced, after which the dies are separated and the moulded lens removed. The complete cycle of operations at the moulding press occupies about ten minutes per lens.

When the lens comes from the press it is in its final form and its surfaces are brightly polished, requiring no further treatment except a hardening operation, in which a fine silica film, a few wave-lengths in thickness, is deposited on the surfaces.

This moulding method of producing finished lenses introduces some interesting points, of which I will mention one. In the case of astigmatic lenses, sphero-cylindrical or toric, two marks or projections at the extremities of the cylinder axis or base curve can easily be left on the dies when they are made. The axis direction is thus incorporated once and for all on the die with extreme precision and consequently the axis direction of every lens is automatically marked to the same high order of accuracy during the moulding operation. It is not necessary to stress the increased accuracy and saving of time thereby introduced in the subsequent "laying-out" operations in the dispensing or prescription house.

§ 6. PLASTICS FOR OTHER OPTICAL ELEMENTS

The second transparent plastic material referred to, Polystyrene, is available for the making of other types of lenses. It belongs to the styrol type of glass-like plastics; it has a refractive index of 1·591 and *V* value 31·2.

From these two materials, Perspex and Polystyrene, corrected lenses are in production. The use of non-spherical surfaces overcomes to some extent the limitation to two materials. Such lenses include those required for binoculars, telescopes and photographic lenses.

All the lenses mentioned, spectacle and otherwise, are manufactured in this country by Combined Optical Industries, Ltd.; the spectacle lenses are distributed by Clement Clarke, Ltd.

DISCUSSION

Mr. E. J. Crundall.

(1) In measuring the contrast sensitivity of the eye through a scratched lens was any account taken of the intensity and direction of the illumination on the scratched lens? To illustrate my point, vision through a dirty motor-car windscreen may be adequate until that windscreen is illuminated by rays of the sun from in front or the side, or by approaching head lights at night. As soon as this direct illumination occurs, vision through the screen becomes much more difficult. Would not the same occur with badly scratched lenses?

(2) Could Mr. Emsley give more information about the pressing of plastic lenses? (e.g. of what material are the dies made and at what temperature and pressure do they operate?) Do the lenses maintain their curves exactly or does some allowance have to be made for contraction when cooling? Does such contraction or other cause lead to waviness? (e.g. the −5·00 meniscus exhibited appeared to be somewhat wavy when viewed by reflection on the inner surface).

(3) Are best form curves obtained, as with glass lenses, by simple flexure, or are aspherical surfaces used?

Author's reply. With reference to the points raised by Mr. Crundall:

(1) In the experiment described there were no glaring sources in the field. The screen under observation was illuminated only by the lamps A and B. As stated by Mr. Crundall, a badly scratched lens would scatter the light from bright sources in the field and consequently would interfere with the wearer's vision and comfort. This was alluded to in the paper.

(2) I am afraid I am unable to give more details than are contained in the paper concerning the kind of steel used for making the dies or the temperature and pressure at which the moulding process is operated. The process is such that the desired final curves are generally accurately obtained, although I suspect there is more difficulty with negative lenses in this connection.

(3) Plastic spectacle lenses are made, at present, in best form, meniscus or toric, as for glass lenses. Aspherical surfaces are not needed, although, as stated in the paper, these are used for corrected lenses for instruments.

DISCUSSION

on

The Resolution of Optical Images

held by the Optical Group 22 *January* 1943

The papers contributed to this discussion included:—

(1) "The Airy Disc Formula for Systems of High Relative Aperture", by H. H. Hopkins (*Proc. Phys. Soc.* **55**, 116).

(2) "Resolution in Dark-ground Illumination", by K. B. E. Merling.

RESOLUTION IN DARK-GROUND ILLUMINATION.

By K. B. E. Merling,

Bland-Sutton Institute of Pathology, Middlesex Hospital, London, W.1.

ABSTRACT. When working with bacteria, bacteriophages and viruses, the limits, first of resolution and then of visibility, were approached. Examples showed how resolution was increased by increasing the numerical aperture (N.A.) and the refractive index. Internal structures, involution granules and spores in dried not-fixed bacteria, bacteriophages from *B. coli* and staphylococci can be resolved in this way. A method of azimuthal illumination was shown to give similar results with active cells. Further, a method was described in which resolution was obtained by reduction of intensity; the reduction was effected by crossing polaroid discs or by interposing neutral grey filters.

This method of increased resolution by decreased intensity was illustrated by a number of photographs showing the separation of otherwise inseparable holes in a screen, the resolution of the shape of otherwise unresolved holes and the application of this method to viruses. In this way the slit between diplococci could be resolved after highest N.A.

and n had not rendered it resolvable. The elementary bodies of certain viruses were separated and their shapes resolved. This method has been applied successfully to the measurement of the sizes of virus bodies. The measurements were checked and confirmed with a microphotometer built on the artificial star principle.

Most of the results were recorded photographically as well as observed visually, showing that the effect of increased resolution by reduced intensity can be produced if the eye is the perceptive instrument or if the photographic emulsion is employed for the purpose. The medium of reception, whether retina or emulsion grain, imposes its modifications on the effect, which was realized with a number of different optical systems by various makers.

To sum up, it appeared in the course of the experiments described that all ways of increasing resolution must lead to the same effect, that is the reduction of the diameter of the Airy disc or diffraction fringe. Whether this is done by increasing the N.A. or n, or decreasing the wave-length or the intensity, the result must be in the same direction. The factors N.A., n and λ are represented in the formula for resolution, and it was suggested that intensity should be included.

(3) "Resolution in Photo-micrography", by J. Smiles.

(4) "The Theory of the Microscope: III, Boundary Waves in Dark-ground Illumination", by L. C. Martin (*Proc. Phys. Soc.* **55**, 104).

GENERAL DISCUSSION

Mr. J. W. Perry. The analysis and interpretation of diffraction images such as those shown, in terms of the form and structure of the immediate " object ", the question raised by Mr. Selwyn, is certainly possible, with some limitations. The problem—which involves the solution of an integral equation such as $I'(x')=\int_{-\infty}^{+\infty} I(x)F(x-x')\,dx$, this applying to the one-dimensional case, $I(x)$ and $I'(x')$ here being the intensity distributions along corresponding lines in the object and image planes, and F being a distribution function subject to $\int_{-\infty}^{+\infty} F(x, x')\,dx'=1$—may be regarded as a generalization of the Paschen-Runge problem of slit-width correction, and its applications in spectrophotometry are of a similar character. The general solution for the case when F represents the intensity distribution in the focal plane resulting from Fraunhofer diffraction in the presence of residual aberrations appears to present merely mathematical difficulties, but the problem is not, of course, entirely theoretical. The chief criticism which might be urged against the method would be the time required to prepare the necessary data and to analyse in this way parts of a photograph, although this would probably not apply in the case of isolated images.

The reference by Mr. Smith to the unpublished work of Michelson on the interpretation of diffraction images reminds one of the well-known problem of light-wave analysis treated by Michelson, and the two subjects are closely allied. Michelson's problem is, in effect, a particular case of the above. The important position which this classic work now holds may justify calling attention to the distinction between resolution, definition and " accuracy " which was made in one of Michelson's earlier papers (*Amer. J. Sci.* 1890, **39,** 139) on this subject. " Resolution " here has the conventional meaning : " definition " denotes immediate recognizability of the form of the object, while " accuracy " denotes the limit of structural form in the immediate object, which is inferrible (as distinct from what is directly perceptible) from interference in the image plane. The methods here involved were different, Michelson's analysis depending upon the variation of visibility with path difference, but the concepts which he found it appropriate to employ have nevertheless undiminished significance in relation to present problems. It seems to me that in this may lie the reason for the dependence of resolution upon light intensity, found by Dr. Merling, always remembering that our measure of resolving power is only a convention.

Dr. R. S. Clay. I suggest that Prof. Martin might be able to obtain a line giving a small phase retardation by the method used to produce a non-reflecting film on a glass surface. With the aid of two razor blades placed on the glass at the desired separation, the film could be restricted to a line of the required breadth, while the retardation could be adjusted by watching the colour of the reflected light.

Prof. R. W. Ditchburn. The results of Prof. Martin's investigation reinforce the arguments in favour of caution in interpreting what is seen in the microscope. In any experiment, it is difficult to say how much of what we see is due to the actual structure of the object, how much to the standard lens aberrations of geometrical optics, how much to diffraction (including the type of effect considered by Mr. Hopkins) and how all this is modified when the light is not monochromatic. When possible it is desirable to check deductions by using objects of known structure (as Prof. Martin has done) and by using monochromatic light.

I agree that the best lens for high contrast resolution is not of necessity the best for low contrast definition, and that by attempting to correct aberrations in the usual way we do not, as a rule, achieve the best result for any one problem. All the same it is probable that a lens corrected in a normal way is likely to be more generally useful than one specially designed to give the very best results for one particular type of object. This does not detract from the value of work done in designing highly specialized lenses when these are required.

Prof. L. C. Martin. My feeling in regard to Dr. Merling's paper is that a good deal of study is necessary before any adequate conclusions can be drawn in respect of his observations. As I have tried to show in the latter part of my own paper, conditions in dark-ground resolution are very special, and the formation of images must differ in important respects from the case of self-luminous objects giving incoherent light.

I have always understood, however, as stated by Mr. T. Smith, that interference effects are independent of intensity as far as observation allows, and I have never been able myself to observe any effects in the microscope which are inconsistent with the interference theory when this is properly applied. But if Dr. Merling has observed effects which he believes are inconsistent he is quite correct in drawing attention to them, though I believe that they will be explicable in terms of physiological contrast effects, coherent illumination, etc., rather than in some unsuspected property of light.

I hope to try Dr. Clay's suggestion for making phase-retarding strips, which might be useful in Zernike's phase-contrast illumination in the microscope.

Prof. Ditchburn's remarks about the optimum correction of lenses puzzle me a little. If it is a question of the best balancing of unavoidable aberration residuals I should agree, but I would say generally that the best corrected lens will give the most readily interpretable image.

AN APPLICATION OF THE CATHODE-RAY OSCILLOGRAPH TO THE MEASUREMENT OF THE WAVE-LENGTH OF SOUND

By G. N. PATCHETT,
Bradford Technical College

Communicated by J. P. Andrews 15 *December* 1942

ABSTRACT. The phase difference between the input to a loud-speaker and the output of a microphone which can be displaced with respect to the loud-speaker is observed. The displacement necessary to change the phase difference by one cycle gives the wave length, and the displacement is adjusted with the aid of a cathode-ray oscillograph.

§ 1. INTRODUCTION

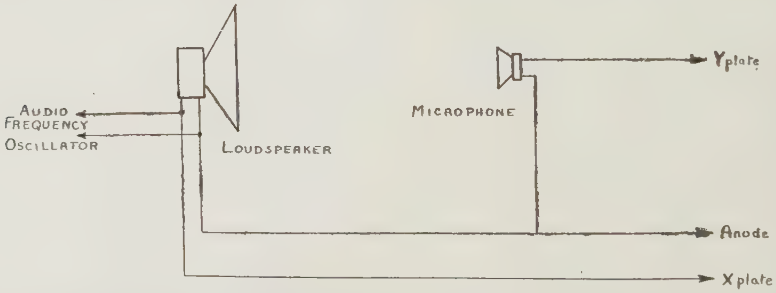
The following description is of a demonstration which illustrates the new experimental methods rapidly coming to the fore in experimental physics, and is particularly interesting as being an easy, yet accurate, measurement very suited to laboratory teaching, whenever a cathode-ray oscillograph may be at hand.

The method is equivalent to measuring the phase difference between the sound emitted at a loud-speaker and the same sound picked up on a pressure microphone a short distance away. The microphone is now moved either from or towards the loud-speaker until the phase difference is again the same, the apparatus used for measuring the phase difference being the cathode-ray oscillograph, either single or double beam.

Under these conditions the microphone has been moved exactly one wave-length.

§ 2. SINGLE-BEAM TUBE

The apparatus is arranged as shown in the figure. The loud-speaker is fed from a suitable audio-frequency oscillator whose output is also fed to the X plates of the oscillograph. The output of the microphone is fed to the Y plates of the oscillograph, if necessary through an amplifier.



The phase difference between the two voltages is shown as an ellipse, circle or straight line on the screen of the oscillograph. When the two oscillations are in phase a straight line is produced (sloping at 45° if the magnitudes of the two voltages are equal).

When the voltages are out of phase (phase difference of 180°), a straight line sloping in the opposite direction is produced. Any other phase difference produces an ellipse or circle.

The microphone is first moved until the two voltages are either exactly in phase (or anti-phase), giving a straight line on the oscillograph. The position of the microphone is noted. The microphone is now moved slowly towards or from the loud-speaker until the *first* position is found where the phase difference is again zero (or 180°), i. e., a straight line in the *same* direction as before.

The distance moved by the microphone now gives the wave-length of the sound at the frequency of the oscillator. It will be noted that as the microphone is moved, the straight line gradually opens out into an ellipse and finally into a circle (if the two voltages are equal in magnitude) at 90° phase difference. Continuing the movement causes the circle to turn into an ellipse sloping the opposite way, and finally into a straight line when the phase difference is 180°.

This repeats itself until the figure finally closes as a straight line with the original slope at 360° phase difference.

§ 3. THE DOUBLE-BEAM OSCILLOGRAPH

The procedure is very similar, except that in this case the phase difference can be seen directly by applying the loud-speaker voltage to one beam and microphone voltage to the other beam of the oscillograph (synchronizing the time base of the oscillograph to the loud-speaker voltage).

In this case movement of the microphone causes a change in the microphone wave-form.

Below are given a few results which have been obtained by this method. These are taken in an ordinary laboratory with no special precautions, the microphone being mounted on a stand and moved along a metre ruler.

Frequency (c/s.)	Measured wave-length (cm.) (corrected to 0° c.)	Calculated wave-length (cm.)
1000	33·25	33·13
4000	8·15	8·3
6000	5·52	5·53

Neither the quality of the loud-speaker nor that of the microphone matters so long as the wave-form is not so distorted as to upset the shape of the ellipse in the single-beam method, or to cause difficulty in detecting when the two voltages are in phase if the second method is used.

An ordinary carbon telephone type of microphone has been found very suitable.

The method gives accurate results, for it does not depend on any phase difference produced by the loud-speaker, microphone or other apparatus, since this will be constant in both cases.

The only precaution needed is to see that the microphone is not upset by reflections, but this is easily prevented by having the loud-speaker and microphone reasonably near each other in a large room.

THE BAND SPECTRUM OF SnSe IN EMISSION

By R. F. BARROW and E. E. VAGO,
Physical Chemistry Laboratory, Oxford

MS. received 3 *February* 1943

ABSTRACT. The band spectrum of tin selenide, previously examined in absorption by Walker, Straley and Smith (*Phys. Rev.* **53,** 140 (1938)), has been excited in emission by a heavy-current uncondensed discharge through the vapour of tin and selenium. A well-developed system of red-degraded bands in the region 3420–4030 A. has been photographed and identified with the system *C* of the above authors. The measurement of some twenty new bands belonging to this system has led to a modification of their v'' assignment and of the ground-state vibrational terms. The following equation, representing the wave-numbers of the band-heads of system *C*, has been derived :

$$\nu_{\text{head}} = 27549\cdot3 + (226\cdot4\,u' - 0\cdot86\,u'^2) - (332\cdot0\,u'' - 0\cdot88\,u''^2)$$

(where $u = v + \frac{1}{2}$). In addition, a number of fainter bands to the short-wave-length side of this system have been measured. These may possibly form a fourth system of SnSe, involving the same lower state, i.e. the ground state, with $\nu_e \sim 30800$ and $\omega'_e \sim 190$.

The absorption by SnSe vapour has been studied by Walker, Straley and Smith (1938), who observed and analysed three systems of bands lying in the range 3690–5390 A. The systems were only incompletely developed, however, and it seemed of interest to examine the spectrum in emission.

A heavy-current uncondensed discharge through a mixture of Al, Sn and Se contained in a silica tube gives rise to a system of roughly evenly-spaced red-degraded bands in the region 3420–4030 A. (see figure 1). The experimental methods have been described in previous papers (Barrow, 1938 and 1941). The band-head measurements are summarized in the Deslandres scheme set out in table 1. The data are represented satisfactorily by the expression

$$\nu_{\text{head}} = 27549\cdot3 + (226\cdot4\,u' - 0\cdot86\,u'^2) - (332\cdot0\,u'' - 0\cdot88\,u''^2)$$

(where $u = v + \frac{1}{2}$), with the exception of the upper state level $v' = 7$, for which residuals of the order of 10 cm.$^{-1}$ suggest perturbation of certain of the associated rotational levels. The vibrational isotope effect is apparent in the increasing diffuseness of the heads with progressive increase of distance from the system-origin, but with the dispersion used (7·4 A. per mm.) it was not possible to identify isotopic heads with any certainty. Even neglecting molecules of abundance less than 10 % of the most abundant molecule, i.e. ^{120}Sn^{80}Se, no fewer than twenty-one other isotopic species make appreciable contributions. These can be grouped into five sets of approximately superposed heads with the following relative intensities and displacement coefficients $(\rho - 1)$: (36, $-0\cdot0195$), (150, $-0\cdot0125$), (225, $-0\cdot0054$), (122, $\pm0\cdot000$), (77, $+0\cdot0052$), referred to ^{120}Sn^{80}Se. On these groups of heads are superimposed the contributions of less abundant molecules. It is not therefore

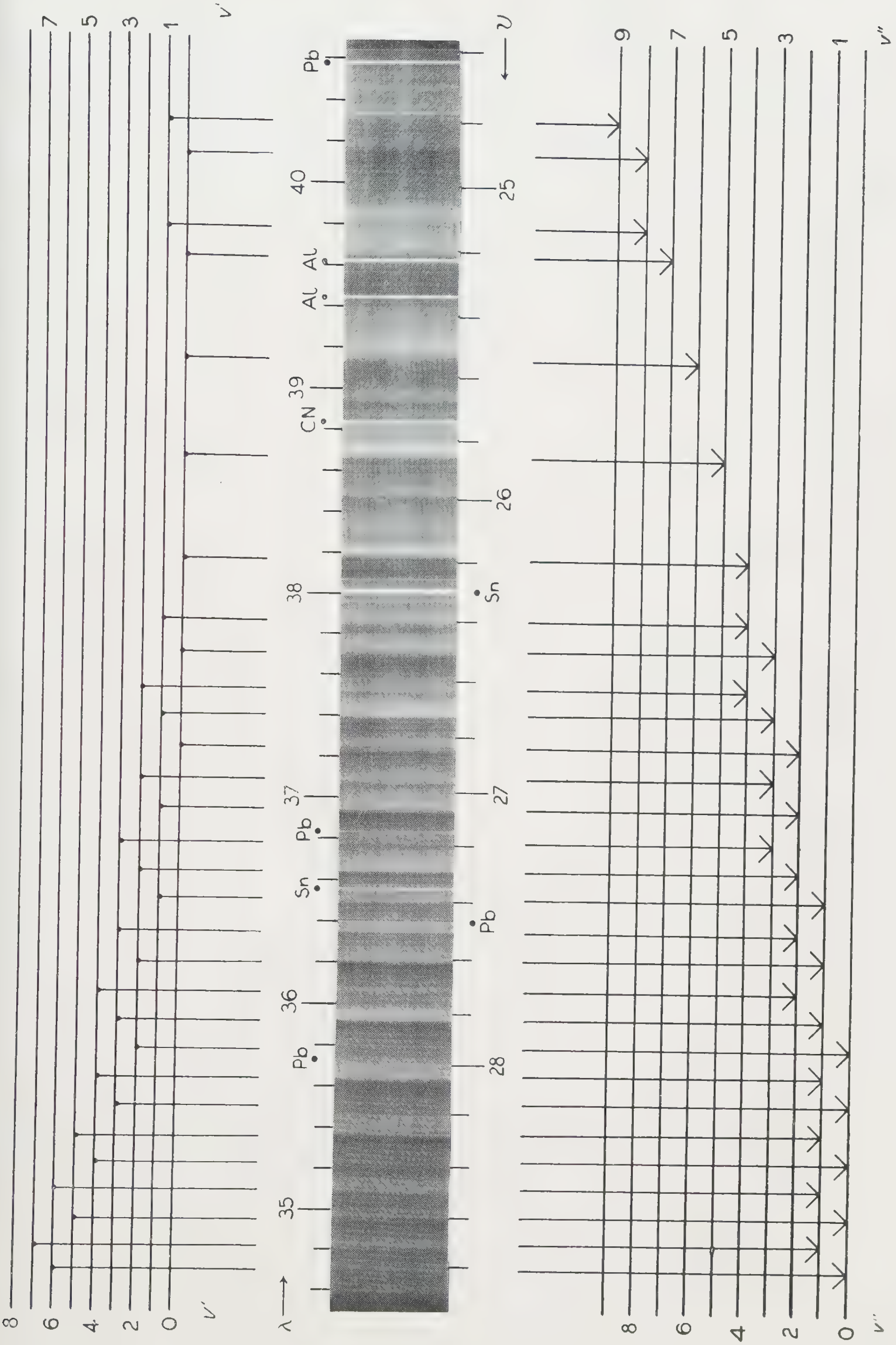


Figure 1. Bands of the C ⇄ X system of SnSe in emission (2·4-m. grating, first order).

To face page 326

Table 1. SnSe system C⇄X band-head data

Italics denotes wave-lengths in air (I.A.);
Large Roman, wave-numbers *in vacuo* (cm.$^{-1}$);
Small Roman, wave-number differences.

* Bands also observed in absorption by
Walker, Straley & Smith.

v' ↑ \ v'' →	0		1		2		3		4		5		6		7		8		9
8	29249·4 *3417·90*	330·8	28918·6 *3456·99*																
	207·9		205·9																
7	29041·5 *3442·26*	328·8	28712·7 *3481·78*																
	221·5		218·3																
6	28820·0 *3468·83*	325·6	28494·4 *3508·46*																
	219·0		224·2																
5	28601·0 *3495·39*	330·8	28270·2 *3536·29*																
	217·6		215·8																
4	28383·4 *3622·18*	329·0	28054·4 *3563·49*	328·8	27725·6 *3605·75*														
	218·4		219·3		218·3														
3	28165·0 *3549·50*	329·9	27835·1 *3591·56*	327·8	27507·3 *3634·37*	321·7	27185·6 *3677·37*												
	223·9		220·4		222·5		218·2												
2	27941·1 *3577·94*	326·4	27614·7 *3620·23*	329·9	27284·8 *3664·00*	327·4	26957·4 *3708·51*	321·8	26635·6 *3753·32*										
			223·8		222·3		221·6		224·3										
1			27390·9 *3649·81*	328·4	27062·5* *3694·10*	326·7	26735·8* *3739·25*	324·5	26411·3 *3785·18*								25126* *3978·8*	323	24803* *4030·7*
					223·3		224·6		225·2								222		
0					26839·2 *3724·84*	328·0	26511·2* *3770·93*	325·1	26186·1* *3817·74*	321·0	25865·1* *3865·12*	323·8	25541·3* *3914·14*	319	25222* *3963·7*	318	24904* *4014·4*		

surprising that with the resolution available, the vibrational isotope effect should fail to provide more than rough qualitative confirmation of the position of the origin-system suggested by the present analysis.

In addition to the bands listed in table 1, a number of similar bands of lower intensity have been measured on the low-wave-length side of this system. Data for these are given in table 2. They do not appear to form part of the main emission system and may belong to a second emission system of SnSe. A tentative analysis gives differences of the order of 325 cm.$^{-1}$ and 190 cm.$^{-1}$,

Table 2. Heads of ultra-violet bands, degraded to the red, observed beyond the band system *C* of SnSe, in discharges through Sn, Al and Se

λ_{air}	ν_{vac}	λ_{air}	ν_{vac}	λ_{air}	ν_{vac}
3434·7	29106	3297·0	30322	3214·0	31105
3397·3	427	82·2	459	03·4	208
74·9	622	67·8	593	3189·7	342
59·8	755	47·4	785	57·6	660
39·0	941	42·2	834	19·3	32049
17·8	30132	27·8	972	02·2	226

Table 3. Comparison between absorption and emission measurements of bands of system *C*

v', v''	Absorption $\nu_{\text{obs.}}$(cm.$^{-1}$)	Emission $\nu_{\text{obs.}}$(cm.$^{-1}$)	B. & V. $\nu_{\text{calc.}}$(cm.$^{-1}$)
1, 2	27062·5	27062·5	27062·5
1, 3	26731·7	26735·8	26735·7
0, 3	517·0	511·2	510·8
0, 4	189·0	186·1	186·1
0, 5	25863·9	25865·1	25862·9
0, 6	541·5	541·3	541·5
1, 7	436·2	—	446·5
0, 7	219·9	222	221·8
1, 8	116·4	126	128·5
0, 8	24901·4	24904	24903·9
1, 9	800·1	803	812·4

Table 4. Summary of spectroscopic data for the isoelectronic molecules SnSe and GeTe

State	SnSe			GeTe		
	ν_e	ω_e	$x_e\omega_e$	ν_e	ω_e	$x_e\omega_e$
? D	(30800)	(190)	—			
C	27549·3	226·4	0·86	27969·5	219·4	1·3
B	22579·5	218·8	0·50			
A	19353·3	223·8	0·88			
X	0	332·0	0·88	? 0	323·4	1·0

and $\nu_e \sim 30800$ cm.$^{-1}$, suggesting that the lower state of the main emission system may also be involved in this system, but further work is necessary to confirm the identity of the emitter and the correctness of the analysis. It is worth noting that a similar set of bands—standing in the same relation to the main emission system—has been observed in the case of SnTe (Barrow, 1940). No bands of the absorption systems *A* and *B* lying on the long-wave-length side of the main emission bands have been observed on the emission plates.

It will be seen from table 1 that ten bands on the low-frequency side of the origin of the main emission system have been observed by Walker, Straley and Smith in absorption. These were assigned to a system *C* by the above authors, together with one additional band at 25436 cm.$^{-1}$, which is not measured in emission. The general agreement between the two sets of observations (see table 3) leaves no doubt of the identity of the emission and the absorption bands. The analysis given in table 1 requires, however, an increase of 2 in the v'' numbers postulated by Walker, Straley and Smith. Their failure to observe bands with $v''=0$ and 1 was presumably due to the presence of overlying impurities, and they state in fact, that heads of Se_2, SeO and SnO systems were present under their conditions. There is no trace of band systems of these molecules on the emission plates. We therefore conclude that the system given in table 1 is identical with the system *C* observed in absorption.

We have now to discuss the ground-state term values. In the equation given above, we have $\omega_e''=332{\cdot}0$, $x_e''\omega_e''=0{\cdot}88$, as compared with the values 333·2 and 1·25 quoted by Walker, Straley and Smith. The former expression fits our measurements of system *C* satisfactorily, with the exception of the outlying band (1, 9): the representation of the strongest bands is quite accurate. This expression is also in fair agreement with the absorption measurements of the bands of this system (table 3), as assigned in table 1. The fit with the band-heads of the absorption system *B* is, if anything, slightly better than that given by the former authors, while the agreement with the stronger bands of system *A* is also reasonably good, except for the (1, 5) and (1, 6) bands of this system, with which residuals of 6·7 and 11·0 cm.$^{-1}$, respectively, are associated. On the whole, however, the expression $G''(v'')=332{\cdot}0(v''+\frac{1}{2})-0{\cdot}88(v''+\frac{1}{2})^2$, which gives much better representation of the new measurements of the levels $v''=0$ to $v''=8$, is to be preferred.

The effect of this alteration on the electronic energies of systems *A* and *B* is slight, but for convenience the data are collected together in table 4—where comparison is also made with the isoelectronic molecule GeTe (Barrow and Jevons, 1940). The close agreement between the electronic energy and vibrational frequencies of the latter molecule with those of system *C* of SnSe makes it seem likely that the lower state of the GeTe transition is also the ground state.

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THE IMPEDANCE OF HOLLOW WAVE GUIDES

By H. T. FLINT and L. PINCHERLE,
King's College, London,
at H. H. Wills Laboratory, University of Bristol

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ABSTRACT. A comparison is made between the treatment of hollow wave guides and conducting guides. It is shown how certain problems in the case of the former can be solved by the familiar methods applied to the latter. In conclusion, suggestions are made for the experimental determination of impedances in hollow guides, particularly for the case of the determination of the impedance of an open-ended guide.

§ 1. INTRODUCTION

MAXWELL'S equations for the electromagnetic field in the space containing a dielectric bounded by a conductor can be expressed in a form identical with the equations which apply to parallel wires or to concentric cables. This leads to the application of the concept of impedance in the case of wave guides similar to that applied in the case of parallel wires or cables.

Schelkunoff (1937, 8) has extended the concept of impedance to a variety of problems, and in particular to the case of wave guides.

It is the purpose of this paper to bring out the close analogy which exists between the two types of guides, so that well known results in one case can be used in the consideration of the other. It will appear in this way that knowledge of the relation between current and potential in the case of parallel guides and of the reflecting properties of impedances placed across them can be used in the solution of questions relating to wave guides.

§ 2. THE BASIS OF THE ANALOGY

Transverse magnetic waves (E-waves)

The characteristic feature of these waves has become well known only in the past few years, although Rayleigh studied the problem as long ago as 1897, at a time when the generation of waves of sufficiently short wave-length to examine his deductions was not possible. In the case of these E-waves the magnetic field is transverse, while the electric field has a transverse and a longitudinal component.

All that is necessary for our present purpose is to show that the equations are reducible to the particular form described.

Maxwell's equations are:

$$\left.\begin{aligned} \operatorname{curl} H &= (Ki\omega + 4\pi\sigma)E/\boldsymbol{c} = (g + i\omega\epsilon)E, \\ \operatorname{curl} E &= -\mu i\omega H/\boldsymbol{c}, \end{aligned}\right\} \qquad \ldots\ldots(1)$$

where σ is the conductivity of the medium in e.s.u. and K is its dielectric constant. The time t is supposed to occur in the factor $e^{i\omega t}$ and the expression $(g+i\omega\epsilon)$ is introduced for the sake of brevity, $g=4\pi\sigma/\boldsymbol{c}$, $\epsilon=K/\boldsymbol{c}$. The conditions in this type of wave can be satisfied by choosing the vector potential $\boldsymbol{A}$, so that the components A_x and A_y vanish while A_z exists, and we shall write $A_z=\mu U$. The scalar potential is denoted by ϕ. Thus, in accordance with the properties of the scalar and vector potentials,

$$\left.\begin{aligned} E_x=-\frac{\partial\phi}{\partial x},\quad E_y=-\frac{\partial\phi}{\partial y},\quad E_z=-\frac{\dot{A}_z}{\boldsymbol{c}}-\frac{\partial\phi}{\partial z}=-\frac{\mu i\omega}{\boldsymbol{c}}U-\frac{\partial\phi}{\partial z},\\ H_x=\frac{\partial U}{\partial y},\quad H_y=-\frac{\partial U}{\partial x},\quad H_z=0.\end{aligned}\right\}\qquad\ldots\ldots(2)$$

It thus follows from Maxwell's equations that

$$\frac{\partial H_y}{\partial x}-\frac{\partial H_x}{\partial y}=-\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)U=(g+i\omega\epsilon)E_z;$$

$$-\frac{\partial H_y}{\partial z}=\frac{\partial^2 U}{\partial z\partial x}=-(g+i\omega\epsilon)\frac{\partial\phi}{\partial x}.\qquad\ldots\ldots(3)$$

This, taken with the third of this group of equations, gives

$$\frac{\partial U}{\partial z}=-(g+i\omega\epsilon)\phi.\qquad\ldots\ldots(4)$$

From equation (3), after substitution of E_z from equations (2) and by making use of equation (4),

$$\nabla^2 U=\mu i\omega(g+i\omega\epsilon)U/\boldsymbol{c}=\tau^2 U,\qquad\ldots\ldots(5)$$

where $$\tau^2=\mu i\omega(g+i\omega\epsilon)/\boldsymbol{c}.$$

It will now be supposed that propagation takes place in the direction of z, so that a solution is sought in which this variable occurs in the factor $e^{\pm Pz}$.

We therefore write $U=Fe^{\pm Pz}$, where F is a function of x and y only. Thus the equation (5) reduces to the form

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)F=(\tau^2-P^2)F=-\chi^2 F,\qquad\ldots\ldots(6)$$

where $$P^2=\tau^2+\chi^2.$$

The expression for U is thus

$$U=F_1 e^{Pz}+F_2 e^{-Pz},\qquad\ldots\ldots(7)$$

where F_1 and F_2 are two functions satisfying equation (6), related by some boundary condition.

As a result of equation (4) it follows that

$$\phi=\frac{P}{g+i\omega\epsilon}(F_2 e^{-Pz}-F_1 e^{Pz}).\qquad\ldots\ldots(8)$$

So far as propagation down the tube is concerned, F_1 and F_2 may be regarded as constants, since they do not depend on z but refer to the distribution of the potentials and field components across the tube.

Equations (7) and (8) may be written in the form

$$U=U_0\cosh Pz-\frac{g+i\omega\epsilon}{P}\phi_0\sinh Pz, \qquad \ldots\ldots(9)$$

$$\phi=\phi_0\cosh Pz-\frac{P}{g+i\omega\epsilon}U_0\sinh Pz, \qquad \ldots\ldots(10)$$

where $U_0\equiv F_1+F_2$ denotes the value of U at the origin $z=0$, and
$\phi_0\equiv P(F_2-F_1)/(g+i\omega\epsilon)$ denotes the value of ϕ at the origin.

From equations (3) and (6), or directly from equations (9) and (10),

$$\frac{\partial\phi}{\partial z}=-\frac{P^2}{g+i\omega\epsilon}U=-[\chi^2/(g+i\omega\epsilon)+\mu i\omega/\boldsymbol{c}]U. \qquad \ldots\ldots(11)$$

Equations (4) and (11) are to be compared with the line equations

$$\frac{\partial I}{\partial z}=-(S+Ci\omega)V, \qquad \frac{\partial V}{\partial z}=-(R+Li\omega)I, \qquad \ldots\ldots(12)$$

where I and V denote the current in the wires and the potential difference between them respectively, $(R+Li\omega)$ is the vector impedance per unit length of the wires and $(S+Ci\omega)$ is the vector admittance per unit length.

The solutions of equations (12) are

$$\left.\begin{aligned} I&=I_0\cosh Pz-\frac{V_0}{Z_0}\sinh Pz,\\ V&=V_0\cosh Pz-Z_0I_0\sinh Pz,\end{aligned}\right\} \qquad \ldots\ldots(13)$$

where I and V denote values at the origin, Z_0 is the characteristic resistance and P is the propagation constant.

$$P^2=(R+Li\omega)(S+Ci\omega), \qquad Z_0=P/(S+Ci\omega)=(R+Li\omega)/P$$

in the case of wires, Z_0 denoting the characteristic impedance of the wires. It is thus suggested, by analogy, that the vector impedance in the case of the tube is

$$X=\chi^2/(g+i\omega\epsilon)+\mu i\omega/\boldsymbol{c} \qquad \ldots\ldots(14)$$

and the vector admittance

$$Y=g+i\omega\epsilon. \qquad \ldots\ldots(15)$$

Thus the characteristic impedance is

$$Z_0=P/Y=X/P. \qquad \ldots\ldots(16)$$

From equations (2) it follows that

$$E_z=\chi^2U/(g+i\omega\epsilon). \qquad \ldots\ldots(17)$$

Equations (9) and (10), when compared with (13), show that the problem of the hollow wave guides can be studied by analogy with that of parallel wires in which the current is U, potential difference ϕ and characteristic impedance Z_0.

The case of an *infinitely long tube* will be considered first. Let it be supposed to be half-filled with a dielectric of constant K_1 and half with one of constant K_2, separated by a plane normal to the sides of the tube. The circuit analogy consists of semi-infinite parallel wires terminated by an impedance equal to the characteristic impedance of the second half of the tube.

According to equations (15) and (16), the wires are to be considered as having a characteristic impedance $Z_0' = P_1/(g_1 + i\omega\epsilon_1)$, and they are terminated with an impedance $Z_0'' = P_2/(g_2 + i\omega\epsilon_2)$. What is usually required at a surface of separation of two media are the coefficients of reflection and of transmission. These are readily obtained from the impedances, since the voltage reflection at an impedance Z, placed across parallel wires, the reflected voltage, V_r, and the incident voltage, V_i, are in the ratio

$$V_r/V_i = (Z - Z_0)/(Z + Z_0),$$

where Z_0 is the characteristic impedance.

The corresponding relation between currents is

$$I_r/I_i = (Z_0 - Z)/(Z + Z_0).$$

In the present case V is replaced by ϕ and I by U, and for reflection at the surface of separation

$$U_r/U_i = (Z_0' - Z_0'')/(Z_0' + Z_0'').$$

As an illustration of this relation let there be no absorption in the dielectrics, so that $g_1 = g_2 = 0$. Since $K_1 = c\epsilon_1$, $K_2 = c\epsilon_2$, the coefficient of reflection becomes

$$(P_1K_2 - P_2K_1)/(P_1/K_2 + P_2K_1).$$

This is a well known relation, and is to be obtained by the use of boundary conditions. The part U_t which is conducted by the impedance and is given by

$$U_t/U_i = 2Z_0/(Z + Z_0)$$

in this case has the value

$$2P_1K_2/(P_1K_2 + P_2K_1).$$

Effect of a dielectric slab placed within the tube

Another problem of interest is that of the effect of a parallel-sided slab of dielectric such as glass or wax placed across the tube.

It is required to consider this in terms of an impedance. Suppose that the tube is infinitely long and that a slab of thickness d of a material of dielectric constant K_2 is placed in it. Let the dielectric constant of the material in the rest of the space be K_1.

Suppose that a wave is incident from left to right. The amount of reflection due to the slab can be calculated by summing up the contributions arising from direct reflection at the first face and from transmission at that face with multiple internal reflections. The total fraction reflected amounts to

$$\frac{(Z_0'^2 - Z_0''^2)\sinh P''d}{(Z_0'^2 + Z_0''^2)\sinh P''d + 2Z_0'Z_0''\cosh P''d},$$

where Z_0' refers to the medium surrounding the slab and Z_0'' and P'' refer to the medium of which the slab is made. The slab and the tube beyond it can be regarded as an impedance of magnitude Z, so that the amount of the reflection is $(Z_0' - Z)/(Z_0' + Z)$.

On equating these expressions it is found that

$$Z = Z_0'' \frac{Z_0' \cosh P''d + Z_0'' \sinh P''d}{Z_0'' \cosh P''d + Z_0' \sinh P''d}. \qquad \text{......(18)}$$

By comparison with the case well known in line theory, it appears that the slab acts as if it were a line of length d and characteristic impedance Z_0'' terminated by an impedance Z_0', i.e. by the characteristic impedance of the tube.

On inserting the values of Z_0' and Z_0''.

$$Z = \frac{P''\mathbf{c}(P'K_2 \cosh P''d + P''K_1 \sinh P''d)}{i\omega K_2(P''K_1 \cosh P''d + P'K_2 \sinh P''d)}. \qquad \text{......(19)}$$

Impedance of hollow tubes with different terminations

The simplest terminations are those in which either U or ϕ vanishes at the end. In the case of parallel conductors the two cases are the open and closed ends in which the current and voltage are zero respectively.

The case $\phi = 0$ corresponds to $V = 0$, and in this case, from equation (2), $E_x = E_y = 0$ at the end. To satisfy this condition the tube must be terminated by a perfect reflector.

As in the case of parallel conductors, it follows that the impedance of a length, l, of tube terminated in this way is

$$Z = Z_0 \tanh Pl. \qquad \text{......(20)}$$

In the case when a tube containing a slab of thickness d is carried on a distance l beyond the slab, and is then terminated with a perfect reflector, the slab and tube beyond it behave like a tube of length d terminated by an impedance $Z_0 \tanh Pl$.

The general formula for the impedance of parallel wires terminated by an impedance Z_r is

$$Z = Z_0 \frac{Z_r \cosh Pd + Z_0 \sinh Pd}{Z_0 \cosh Pd + Z_r \sinh Pd}. \qquad \text{......(21)}$$

Applying this to the case of the tube, the impedance offered by the slab and tube beyond it is

$$Z = Z_0'' \frac{Z_0' \tanh P'l \cosh P''d + Z_0'' \sinh P''d}{Z_0'' \cosh P''d + Z_0' \tanh P'l \sinh P''d}. \qquad \text{......(22)}$$

If the reflection from the slab terminated in this way is required, it can be obtained from the formula

$$-R = (Z_0' - Z)/(Z_0' + Z).$$

After substituting the value of Z from equation (22), we obtain,

$$R = \frac{(Z_0'^2 - Z_0''^2) \sinh P''d + e^{-2P'l}\{(Z_0'^2 + Z_0''^2) \sinh P''d - 2Z_0'Z_0'' \cosh P''d\}}{(Z_0'^2 + Z_0''^2) \sinh P''d + 2Z_0'Z_0'' \cosh P''d - e^{-2P'l}(Z_0'^2 - Z_0''^2) \sinh P''d}. \qquad \text{......(23)}$$

In the case in which there is no absorption this becomes

$$R = \frac{(P'^2K_2^2 - P''^2K_1^2)\sinh P''d + e^{-2P'l}\{(P'^2K_2^2 + P''^2K_1^2)\sinh P''d - 2P'P''K_1K_2\cosh P''d\}}{(P'^2K_2^2 + P''^2K_1^2)\sinh P''d + 2P'P''K_1K_2\cosh P''d - e^{-2P'l}(P'^2K_2^2 - P''^2K_1^2)\sinh P''d} \qquad \ldots\ldots(24)$$

§ 3. TRANSVERSE ELECTRIC WAVES (H-WAVES)

This case is similar to that of the E-waves, the places of the electric and magnetic components being reversed. Thus the relations (2) are replaced by

$$\left.\begin{aligned} E_x &= -\frac{\partial\psi}{\partial y}, \qquad E_y = \frac{\partial\psi}{\partial x}, \qquad E_z = 0, \\ H_x &= -\frac{\partial F}{\partial x}, \qquad H_y = -\frac{\partial F}{\partial y}. \end{aligned}\right\} \qquad \ldots\ldots(25)$$

It will appear that ψ and F replace the potential and current of conducting guides.

As before, it follows that

$$\frac{\partial\psi}{\partial z} = -\mu i\omega F/\mathbf{c} \qquad \ldots\ldots(26)$$

and

$$H_z = -(g + i\omega\epsilon)\psi - \frac{\partial F}{\partial z}. \qquad \ldots\ldots(27)$$

The equation satisfied by ψ is

$$\nabla^2\psi = \tau^2\psi \qquad \ldots\ldots(28)$$

(cf. equation (5)).

The result analogous to equation (17) is

$$H_z = \chi^2\mathbf{c}\psi/\mu i\omega, \qquad \ldots\ldots(29)$$

and the equations relating ψ and F become

$$\left.\begin{aligned} \frac{\partial\psi}{\partial z} &= -\mu i\omega F/\mathbf{c}, \\ \frac{\partial F}{\partial z} &= -(g + i\omega\epsilon + \chi^2\mathbf{c}/\mu i\omega)\psi. \end{aligned}\right\} \qquad \ldots\ldots(30)$$

Thus the tube can be compared with a parallel conducting guide carrying a current F with a potential difference ψ, in which the characteristic impedance is Z_0, where

$$Z_0 = \mu i\omega/P\mathbf{c} = P/g + i\omega\epsilon + \chi^2\mathbf{c}/\mu i\omega. \qquad \ldots\ldots(31)$$

The solution of equations (30) is, as in the previous case, given by

$$\left.\begin{aligned} \psi &= \psi_0\cosh Pz - Z_0F_0\sinh Pz, \\ F &= F_0\cosh Pz - \frac{\psi_0}{Z_0}\sinh Pz. \end{aligned}\right\} \qquad \ldots\ldots(32)$$

Reflection at the boundary separating two dielectrics

As in the example in the case of E-waves, suppose that the tube is infinitely long but half-filled with two substances of dielectric constants K_1 and K_2 and of permeabilities μ_1 and μ_2 respectively. This case can be compared with a pair of parallel wires of characteristic impedance $Z_0'=\mu_1 i\omega/\boldsymbol{c}P_1$ terminated with an impedance $Z_0''=\mu_2 i\omega/\boldsymbol{c}P_2$. Thus the fraction of voltage, ψ, reflected is given by

$$\psi_r/\psi_i=(Z_0''-Z_0')/(Z_0''+Z_0')=(P_1\mu_2-P_2\mu_1)/(P_1\mu_2+P_2\mu_1).$$

Reflection at a dielectric slab

If the slab is parallel-sided and divides an infinite hollow guide into two parts, it behaves like a tube of length d terminated by an impedance Z_0'. Thus the tube behaves as if it were terminated by an impedance, given by expression (18), with the appropriate values for Z_0', Z_0'' and P''.

The value of this impedance is

$$Z=\frac{\mu_2 i\omega(\mu_1 P''\cosh P''d+\mu_2 P'\sinh P''d)}{\boldsymbol{c}P''(\mu_2 P'\cosh P''d+\mu_1 P''\sinh P''d)} \qquad \ldots\ldots(33)$$

and the reflecting power is

$$\frac{(P'^2\mu_2^2-P''^2\mu_1^2)\sinh P''d}{(P'^2\mu_2^2+P''^2\mu_1^2)\sinh P''d+2P'P''\mu_1\mu_2\cosh P''d}.$$

Reflection at a slab terminated by a hollow tube of length l with a perfectly reflecting end

The condition at the end is $\psi=0$, i.e. $H_z=0$, so that the impedance of the length, l, of the tube is the same as that of a parallel conductor guide short-circuited at the end, i.e. $Z_0\tanh Pl$. The slab and the tube beyond it thus present an impedance

$$Z=Z_0''\frac{Z_0'\tanh P'l\cosh P''d+Z_0''\sinh P''d}{Z_0''\cosh P''d+Z_0'\tanh P'l\sinh P''d}. \qquad \ldots\ldots(34)$$

Thus the reflection coefficient, $R=(Z-Z_0')/(Z+Z_0')$, has the value

$$R=\frac{(Z_0''^2-Z_0'^2)\sinh P''d+e^{-2P'l}\{(Z_0''^2+Z_0'^2)\sinh P''d-2Z_0'Z_0''\cosh P''d\}}{(Z_0''^2+Z_0'^2)\sinh P''d+2Z_0'Z_0''\cosh P''d+e^{-2P'l}(Z_0''^2-Z_0'^2)\sinh P''d}$$

$$=\frac{\begin{array}{l}(\mu_2^2P'^2-\mu_1^2P''^2)\sinh P''d\\ \qquad +e^{-2P'l}\{(\mu_2^2P'^2+\mu_1^2P''^2)\sinh P''d-2\mu_1\mu_2P'P''\cosh P''d\}\end{array}}{\begin{array}{l}(\mu_2^2P'^2+\mu_1^2P''^2)\sinh P''d+2\mu_1\mu_2P'P''\cosh P''d\\ \qquad +e^{-2P'l}(\mu_2^2P'^2-\mu_1^2P''^2)\sinh P''d.\end{array}} \qquad \ldots\ldots(35)$$

It will be seen by comparison with equation (24) that this formula for H-waves can be obtained from that for E-waves by replacing K by μ.

These examples suggest methods for determining impedances in the case of hollow guides by analogy with methods which have been applied in the case of conducting guides. From the expressions given it is then possible to deduce values of the dielectric constant and absorption in the case of materials which can be introduced into the guide.

In the case of E-waves, the quantity U which has been compared to current is measured by the component E_z (equation (17)). If it is possible to measure E_z, for example by introducing a short conductor into the tube to act as a probe, the impedance offered by the open end of a tube can be determined. Suppose that U is measured in this way at a convenient origin where the values of U and ϕ are U_0 and ϕ_0, and at the open end where the impedance is Z. If l denote the distance of the end from the origin, it follows from equations (9) and (10) and from the relation $\phi = ZU$, which is in accordance with the definition of impedance, that

$$(Z_0 \cosh Pl + Z \sinh Pl)U = Z_0 U_0.$$

The ratio U_0/U is measured in the experiment so that Z can be determined by a method similar to that described for conducting guides (Flint and Williams, 1941).

Another method suggests itself which does not involve the use of probes, which may upset the field distribution. This consists in introducing a slab of a dielectric of known dielectric constant and thickness d at a distance l from the open end. The analogous case in conducting guides is represented in the diagram (figure 1).

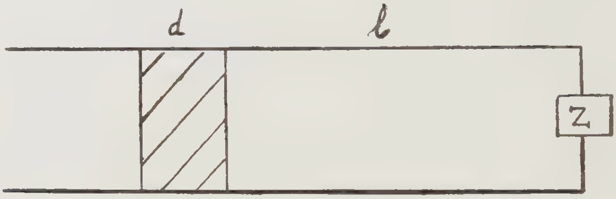

Figure 1.

Z denotes the impedance of the open tube.

The length, l, terminated by Z offers an impedance

$$Z_0' \frac{Z \cosh P'l + Z_0' \sinh P'l}{Z_0' \cosh P'l + Z \sinh P'l},$$

using the same notation as before.

Thus the slab behaves as a tube of length d and of characteristic impedance Z_0'', terminated by the above impedance. Thus the impedance at the end of the hollow tube terminated by the slab and open end is given by

$$Z_0'' \frac{\dfrac{Z \cosh P'l + Z_0' \sinh P'l}{Z_0' \cosh P'l + Z \sinh P'l} Z_0' \cosh P''d + Z_0'' \sinh P''d}{Z_0'' \cosh P''d + \dfrac{Z \cosh P'l + Z_0' \sinh P'l}{Z_0' \cosh P'l + Z \sinh P'l} Z_0' \sinh P''d}.$$

The condition for no reflection back along the tube from this termination is obtained by placing this expression equal to Z_0'. From this follows the relation

$$Z = Z_0' \frac{Z_0' Z_0'' \cosh P''d (\cosh P'l - \sinh P'l) + (Z_0'^2 \sinh P'l - Z_0''^2 \cosh P'l) \sinh P''d}{Z_0' Z_0'' \cosh P''d (\cosh P'l - \sinh P'l) + (Z_0''^2 \sinh P'l - Z_0'^2 \cosh P'l) \sinh P''d}. \qquad \ldots\ldots(36)$$

The reflection coefficient of the open end is of particular importance. From (36) we deduce:

$$R=\frac{Z-Z_0'}{Z+Z_0'}=\frac{e^{2P'l}(Z_0''^2-Z_0'^2)\sinh P''d}{(Z_0''^2+Z_0'^2)\sinh P''d-2Z_0'Z_0''\cosh P''d}, \qquad \ldots\ldots(37)$$

a formula which can, of course, be found also by the use of boundary conditions.

Writing $R=R_0e^{i\phi}$ and assuming that there is no absorption, so that P' and P'' are pure imaginaries, $P'=i\beta'$, $P''=i\beta''$, we find that R_0 is a function only of d (not of l):

$$R_0=\sqrt{\left(\frac{(Z_0''^2-Z_0'^2)^2\tan^2\beta''d}{(Z_0''^2+Z_0'^2)^2\tan^2\beta''d+4Z_0'^2Z_0''^2}\right)}, \qquad \ldots\ldots(38)$$

while ϕ is determined by

$$\cos(2\beta'l-\phi)=R_0\frac{Z_0''^2+Z_0'^2}{Z_0''^2-Z_0'^2}. \qquad \ldots\ldots(39)$$

The experiment will then consist in adjusting d and l until no reflection is produced by the system (slab + end); in these conditions R_0 and ϕ are immediately given by (38) and (39). Once R_0 and ϕ are known for a given tube, a slab of a material of unknown dielectric constant can be substituted, and we have then a null method for the determination of this quantity.

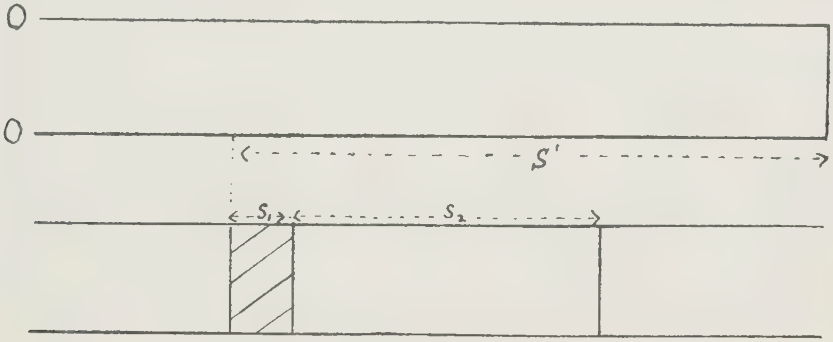

Figure 2.

As a final example of the application of these methods we consider a case which has been suggested by Lamont (1941) and which forms the basis of a method for the determination of dielectric constants suggested by him.

A hollow tube in which H-waves are generated is closed at one end by a perfect reflector and distances are measured from an origin chosen at a node.

On placing a dielectric slab within the tube, the reflector has to be displaced in order to restore conditions similar to those of the first case, i.e. to restore resonance.

This means that the impedance of the length s' of the empty tube, i.e. $Z_0'\tanh P's'$, is equal to that of the tube containing the slab terminated by the tube of length s_2. Thus

$$Z_0'\tanh P's'=Z_0''\frac{Z_0'\tanh P's_2\cosh P''s_1+Z_0''\sinh P''s_1}{Z_0''\cosh P''s_1+Z_0'\tanh P's_2\sinh P''s_1}.$$

For *H*-waves $Z_0 = i\omega/cP$ with $\mu = 1$, and in the problem considered there is no absorption, so that

$$P = i\beta = 2\pi i/\lambda.$$

Thus $$Z_0'/Z_0'' = \lambda'/\lambda'' = n \text{ (say).}$$

The above equation then reduces to

$$n \cot \beta'' s_1 (\cot \beta' s' - \cot \beta' s_2) + \cot \beta'' s_1 \cot \beta' s_2 + n^2 = 0.$$

This is the relation obtained by Lamont, which forms the basis of calculation in his experiment.

N.B.—The numerical value of the characteristic impedance in the case of no absorption may be obtained from the formulae $Z_0 = 377\sqrt{(1-\eta^2)}$ ohms for *E*-waves and $377/\sqrt{(1-\eta^2)}$ ohms for *H*-waves, where $\eta = \lambda/\lambda_c$, λ_c being the cut-off wave-length.

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A COMPARISON OF THE MAGNETIC PROPERTIES OF SOLID AND POWDERED MAGNETITES: WITH OBSERVATIONS ON COERCIVE FORCE

By E. F. HERROUN,
Reigate

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§ 1. INTRODUCTION

In connexion with an endeavour to measure the susceptibility of ferromagnetic ferric oxide and other compounds of high susceptibility, a paper was published (Herroun and Wilson, 1928) in which a method was described of mixing the powders with a binding material, such as a warm solution of gelatine, and compressing the stiff paste in a gun-metal mould so as to form rectangular bars, which could be used in the isthmus method in the Stalloy ring magnet previously used for cut bars of magnetite, described in detail in an earlier paper on magnetite (Wilson and Herroun, 1919).

Realizing that such a method would not result in the formation of a continuous solid, some magnetites were ground into powder and treated in the same way in order to find the effect of the process on known materials previously measured in the form of bars cut from the solid. The magnetites then used were from Mineville and from the Tilly Foster Mine, both in New York State;

both of these magnetites possessed relatively high susceptibility and low coercivity. The results showed that powdering and reconstructing bars of built-up material resulted in a great reduction in permeability and an increase in coercivity, with, however, a reduction in remanent magnetism.

In more recent papers by V. H. Gottschalk and C. W. Davis (1935) on "The Influence of Grain Size in Magnetites" these authors state that coercive force increases as grain size diminishes, and pronounce that "coercive force is directly proportional to specific surface, that is to the square centimetres of surface per gram of powder". They also say that "this proof is so direct that this simple relation may be expected to hold generally, not only in the case of other minerals, but also in powders of electrolytic iron, permalloy and other ferromagnetic alloys used in compressed magnet cores as well as for the dispersed particles that cause the hardness of alpha-iron magnets".

Their experiments appear to have been conducted on two kinds of magnetite, viz. Mineville and Hayden Slag magnetite. Both of these belong to the class of high-permeability and low-coercivity magnetites, but the wide generalization given above is hardly justified unless other materials of high initial coercivity fall into line with their general statement.

2. EXPERIMENTAL

A number of magnetites of high, medium and low coercivity have been examined, and the results for bars cut from the solid mineral compared with those for coarse- and fine-grain powders prepared from the same magnetites are given in tables 1 and 2.

The bars are 4 cm. long by 1 cm. × 1 cm. in section and fit into the recesses in the poles of the Stalloy magnet which has been fully described in the 1919 paper already referred to.

These bars can be easily removed and used for determining J_{rem} by means of a magnetometer as well as for finding the coercivity by reducing J_{rem} to zero by adjusting an adverse field in a solenoid.

The bars of powder were formed by mixing the powder with a small quantity of a hot solution of photographic gelatine to form a paste of putty-like consistency and compressing this in a gun-metal mould under a pressure of about 100 kg. per sq. cm. When cold, the rectangular bar is removed and allowed to dry in air.

Gottschalk and Davis employ brass tubes with walls 0·9 mm. thick and wind the secondary coil directly over the brass tube, and although aware of the objection that might be raised to such an arrangement, they claim that it only alters the phase of the induced current, the ultimate deflection being the same.

In comparing the powders compressed into solid bars with a binding material with the dry powder in tubes, and to avoid delay in the circuit of the ballistic galvanometer, Bakelite tubes fitted with soft iron plugs were employed, the secondary winding being wound over the central position, which only contained the powder. The results obtained with Arkansas, Mineville and Russian magnetites in the case of the dry powder when compared with the compressed bars differed in no important respect, except that the density was in all cases less, as would naturally be expected, so the bar method was generally adopted. The galvanometer employed had a sensitivity such that a steady deflection of

1 mm. at 1 metre represented a current of $1{\cdot}045 \times 10^{-8}$ ampere, and its periodic time was 7·3 seconds.

Table 1. Magnetic properties of solid magnetites

Locality	Density	H for μ_{max}.	μ max.	H max.	B_{rem} %	H_c for $B=0$	H_c for $\mathcal{J}=0$	$\mathcal{J}_{\text{rem}}$ after $H=1200$ Oe.
Arkansas, U.S.A.	4·78	290	5·5	318	37·5	102	190	61·5
Penryn, Cornwall	4·59	220	4·95	320	25·5	82·5	147	36·8
Tilly Foster Mine, N.Y., No. 1	4·1	36	19·3	318	14·4	9·4	18·5	4·2
Tilly Foster No. 2	4·75	40	34·6	318	12·2	7·8	15·5	2·85
Mineville, N.Y.	5·1	55	34·9	259	23·8	20·7	26·2	8·85
Russia: locality unknown	4·72	150	11·55	300	20	69	101	38·4
Unknown locality	4·675	96	16·8	318	38	52·7	75	24·6

Notes to table 1

Little comment is called for regarding the first five columns except that the last magnetite in the list, from an unknown locality, was included because it gave practically identical values for coercivity in the form of a solid bar and in a bar of fine powder. It was obtained in 1922 from Messrs. Harrington Brothers, who, however, are unable to give any information respecting the locality of origin. It is a strong compact magnetite of moderately high permeability with a coercivity intermediate between the low value of Tilly Foster and the high value of Arkansas magnetites.

The column " B_{rem} % " is calculated in the usual way, by subtracting the deflection due to " break " from half the deflection on reversal and dividing by the latter and multiplying by 100 to represent the percentage.

The great difference between H_c for $B=0$ and H_c for $\mathcal{J}=0$ is naturally due to the fact that when H_c in the ballistic method has been adjusted until the deflection from $+H_{\text{max.}}$ to $-H_c$ is exactly half that due to the total reversal, the specimen is still under the influence of a demagnetizing force which, on breaking the primary current, disappears and allows it to regain a considerable fraction of its former magnetization, as shown by the reverse deflection of the galvanometer. H_c for $\mathcal{J}=0$ is obtained by placing the bar in the middle of a solenoid and increasing the demagnetizing current step by step until the specimen is completely neutral to a magnetometer needle when no current is acting.

§ 3. GENERAL CONCLUSIONS

From a comparison of tables 1 and 2 it will be seen that the effects of powdering may be summarized as follows:

1. Permeability is much reduced by powdering, the reduction being greater in proportion for magnetites of higher initial permeability.

2. Maximum permeability is reached at higher forces with powders than with solid.

3. Coercivity is increased in the case of magnetites of low initial coercivity, but decreased with those of high coercivity, while for some of intermediate value there is very little change.

Table 2. Magnetic properties of powdered magnetites

Description	Density	H for $\mu_{\text{max.}}$	μ max.	H max.	B_{rem} %	H_c for $B=0$	$\mathcal{J}_{\text{rem}}$ after $H=1200$ Oe.
Arkansas:							
Fine-grain bar	3·24	300	3·74	318	23·6	96	32
Powder in tube	2·747	308	3·2	308	23	74	—
Penryn:							
Fine-grain bar	2·66	154	2·72	318	14·4	60·4	15·8
Tilly Foster, No. 1 :							
Coarse-grain bar	2·84	69	4·4	318	9·9	20·4	3·75
Fine-grain bar	2·56	81	3·2	318	9·0	20·75	3·4
Tilly Foster, No. 2 :							
Coarse-grain bar	3·15	69	8·65	318	8·7	17·3	4·25
Fine-grain bar	3·06	96	6·58	318	8·7	18·5	6·15
Mineville:							
Mixed-grain bar	3·45	96	9·3	318	17·4	31	15·3
Coarse-grain bar	3·125	96	7·1	318	11·3	21	7·1
Fine-grain bar	3·206	116	7·8	318	18·5	35	14·7
Fine-grain in tube	2·74	78	6·9	308	18·2	30	—
50% *Arkansas* 50% *Mineville*:							
Mixed-grain bar	3·26	155	5·1	318	21	56·4	27
Russia:							
Coarse-grain bar	2·76	195	4·8	318	19·3	48	14·7
Coarse grain in tube	2·637	195	4·4	308	19·8	55	—
Fine-grain bar	2·91	190	4·55	318	19·2	56	16·7
Fine grain in tube	2·597	200	4·3	308	18·7	64·5	—
Unknown locality :							
Coarse-grain bar	2·92	154	5·66	318	17·8	52	13·6
Fine-grain bar	2·72	154	5·8	318	14	50·5	12·3

Notes to table 2

In the earlier publication (1928) the magnetites were reduced to fairly fine powder, but were not separated by a sieve, and were incorporated into bars with a binding material. These are described as " mixed-grain size ". It will be noticed that these mixed bars show a higher density than either the " coarse "- or " fine "-grain bars, and in consequence a higher value for μ.

The " coarse grain " used showed a grain size lying between 0·2 and 0·05 mm. and the " fine grain " from 0·05 to 0·02 mm.

In the column giving H for $\mu_{\text{max.}}$ the results must be regarded as approximate only, as with these powder bars the μ-H curve is very flat-topped, and it is difficult to be very precise as to where μ reaches its maximum value.

The bar of mixed 50% Arkansas and 50% Mineville magnetites is interesting in that it gives values for both H_c and retained magnetism not far from the mean of those for the constituents, while it might have been expected to lower the high coercivity Arkansas to nearly the low value for Mineville.

No attempt was made to measure $\mathcal{J}_{\text{rem}}$ after exposure to a field of 1200 Oe. in the case of powders in tubes, owing to their probable instability and to the effect of the soft iron plugs in the ends of the tube : the compressed bars were treated in the same way as the cut bars when used in the magnetometer.

There was one magnetite, from the Hey Tor Mine, South Devon, which, although of low coercivity (2·38 after $H_{\text{max.}}=525$: see 1919 paper), when powdered and examined in the usual way showed no increase in coercivity. This has not been included in the tables for the following reason. Many magnetites contain small inclusions of non-magnetic minerals, but the Hey Tor specimen consisted of a mass of very small crystals of magnetite bonded together by silicates, so the subdivision already existed in the natural mineral, and after powdering and sifting it was found that the fine-grain powder contained a greater percentage of magnetite and the coarse grain a greater proportion of mixed silicates than the original rock. This specimen shows that fine division in the natural product is not associated with high coercivity.

On the other hand, the most compact and uniform magnetite met with, which also possesses the lowest electrical resistance (0·590 ohm. per c.c., W. & H., *Proc. R.S.* 1924) and has the highest coercivity (150 for $H_{\text{max.}}=525$: see 1919 paper), is that from Arkansas.

In Gottschalk's paper already referred to he quotes results obtained by Sizoo (*Z. Phys.* **51**, 557, 1928) for pure iron and nickel. The latter found an increased coercivity with iron from $H_c=0{\cdot}388$ to 0·754 as the grain size diminished from 11 mm. to 0·1 mm., and for nickel from $H_c=0{\cdot}45$ to 1·26 as the grain size decreased from 2 mm. to 0·3 mm.

Both these pure metals have remarkably small coercivity, and it would be of great interest if a high-coercivity steel could be reduced to powder without either an alteration due to heating, or oxidation due to very fine division, and compared with the original solid. This, however, presents great difficulty.

The reduction of magnetic flux for a given field, due to the diminution of permeability, is probably an important factor in increasing coercivity. No material of high coercivity has at the same time a high permeability, but low permeability in itself is not necessarily associated with high coercivity, as, for example, in ilmenite (free from magnetite), where the maximum permeability is only 1·47 for a field of 500 Oe., yet its coercivity is only 25·9 after a field of 1165 Oe.

§ 4. NOTES ON THE PERSISTENCE OF MAGNETIC STRUCTURE

In the 1919 communication to this Society on magnetite an account is given of a bar of Penryn magnetite which, after being heated to above 585° c. (the Curie point for magnetite), was allowed to cool in a vertical position in the earth's field. It was found, when cold, to have acquired a remanent magnetism which would have required a force of 94 Oe. at room temperature to impart.

J. G. Koenigsberger in an article on "The stability of thermoremanence in rocks, etc." in 1938 remarks that a rock, like basalt, containing magnetite may acquire a J_{rem} value of 100 to 1000 times as great as that due to the earth's field at 20° c.

It has been known for about a century that powerful magnets could be made by cooling from a red heat steel bars placed across the poles of a powerful magnet. But the fixity of the magnetization thus imparted seems to be not generally known.

When a bar of magnetite has been magnetized by cooling from about 600° c. in a strong magnetic field it requires a large adverse field to reduce its magnetic moment to zero. If, however, this is carefully done, and the bar exhibits no residual magnetism, it will be found that any adverse field *less* in magnitude than that required to demagnetize it produces a small temporary magnetization which reverts to zero on switching off the magnetizing current. If now the current be reversed, so that it is in the direction required for its original polarity, not only is the temporary magnetization much greater, but on again breaking the circuit the bar will be found to have recovered about half its original J_{rem}. The purely temporary and smaller effect of the adverse field compared with the direct or positive field results in the fact that an alternating current in the solenoid, controlled so that its maximum (not root-mean-square) does not exceed the force used to demagnetize it, will result in a considerable restoration of its original J_{rem}. Naturally, if a sufficiently large alternating field be employed, the bar can be demagnetized in the usual way, but with a limited current there is the paradox of an apparently neutral bar being magnetized by an alternating current reduced to zero.

That this result is not peculiar to magnetite has been proved by using a bar of "self-hardening steel" treated in the same manner, when it reproduced the effects noticed with magnetite.

The explanation would appear to be due to the persistence of molecular or "domain" grouping, and is rendered probable by the fact that a similar, but usually less marked, result can be obtained in the cold by submitting the bar to a very intense field and then demagnetizing carefully to neutrality by an adverse field, when it will generally show a pronounced bias in favour of the direction of strong magnetization.

This effect of hidden or latent magnetization is clearly different from the increase in $BH_{\text{max.}}$ in hard magnet steels by cooling from 1200° in an intense magnetic field of 4400 Oe. as described by D. A. Oliver and J. W. Shedden (1938), as the former effect is easily manifested in magnetite, or even in rocks containing magnetite, when cooled from 600° C. in the earth's field, although probably the actual mode of causation may be similar.

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REVIEWS OF BOOKS

The Transition from War to Peace (Oxford Pamphlets on Home Affairs, No. H 3), by A. C. Pigou. Pp. 31. (Oxford: The University Press, 1943.) 6*d*. net.

It is not for a physicist as such to express an opinion on matters of economics, but as a citizen he is entitled to do so ; his training in the logical ordering and interpretation of facts should enable him to form a sound judgement, and should safeguard him from reaching his conclusions by emotional rather than rational channels; but this very habit of mind will make him insist on as much data as he can gather before he reaches any conclusion. It is only in so far as he follows these principles—importing his scientific attitude into these other regions—that there can be any justification for acceding to the request, put forward from time to time by some scientists (and these not all meeting the condition set out above), that scientists should as a class be granted a bigger share than other members of the community in the management of affairs.

These reflexions are induced by reading Professor Pigou's pamphlet. This is an example of the application of reason to an economic problem, rather than a handbook of data on which to base reasoning, and ought, like Euclid's or any other chain of logical deduction, to be checked by attentive readers on the look-out for possible hidden assumptions.

It is one of a series, which will deal with Trade Unions, Education, Money, the National Debt and many other subjects ; three of them have already been published, entitled respectively : *How Britain is Governed, The Newspaper,* and *Britain's Future Population.* Though rather brief, they will all be by authoritative writers, and should not be overlooked by those in search of short accounts of some of the problems of the present day. J. H. A.

Bibliography of the Literature Relating to Constitutional Diagrams of Alloys, by J. L. Haughton, D.Sc. Pp. iv+63. (London: The Institute of Metals, 1943.) 3*s*. 6*d*.

This publication provides a convenient approach to the task, previously a very laborious one, of examining the existing knowledge on the constitutional diagram of an alloy system. The book under review, being a bibliography, is restricted to lists of relevant references, but a very useful feature is that reference is made to the Institute of Metals Abstract, where there is one, as well as to the original paper. The references given are up to date (including some for 1942), and apparently complete. Dr. Haughton and the Institute of Metals are to be congratulated on their enterprise in producing this valuable bibliography. B. C.

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Natural and Physical Sciences. (Catalogue 464, 1943.) Pp. 44. William George's Sons, Ltd., 89 Park Street, Bristol, 1. 3*d*.

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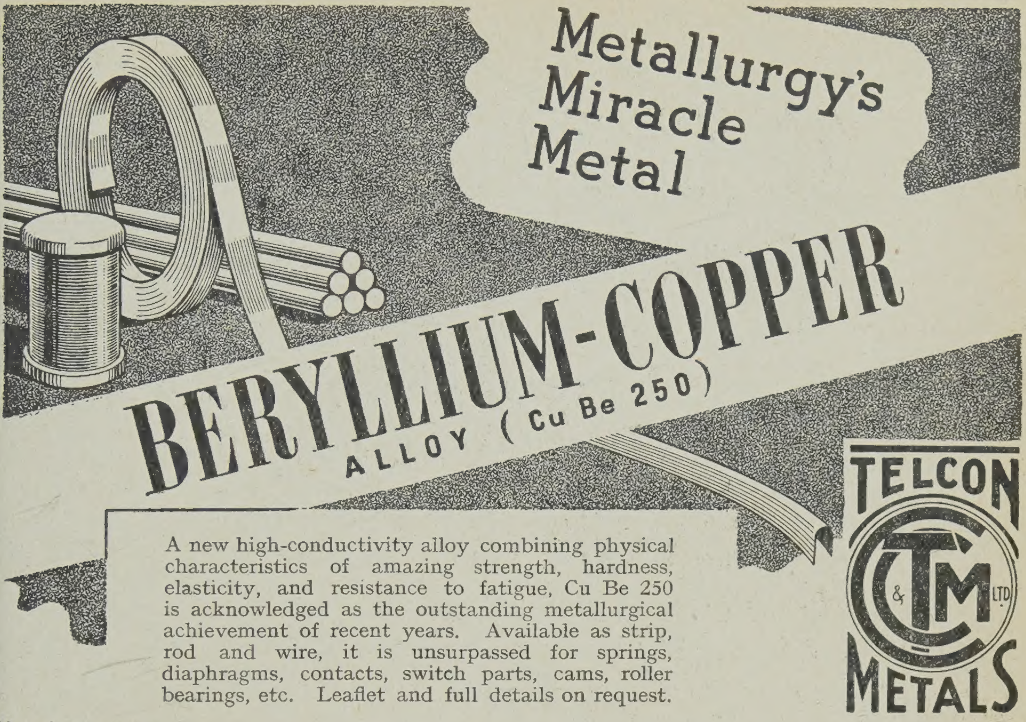

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